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Spontaneous Lorentz Breaking at High Energies

Hsin-Chia Cheng [a,b], Markus A. Luty [a,c,d], Shinji Mukohyama [a,e] and Jesse Thaler [a]

[a] *Jefferson Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138*

[b] *Department of Physics, University of California, Davis, California 95616*

[c] *Physics Department, Boston University, Boston, Massachusetts 02215*

[d] *Physics Department, University of Maryland, College Park, Maryland 20742*

[e] *Department of Physics and Research Center for the Early Universe, The University of Tokyo, Tokyo 113-0033, Japan*

Abstract

Theories that spontaneously break Lorentz invariance also violate diffeomorphism symmetries, implying the existence of extra degrees of freedom and modifications of gravity. In the minimal model ("ghost condensation") with only a single extra degree of freedom at low energies, the scale of Lorentz violation cannot be larger than about $M \sim 100$ GeV due to an infrared instability in the gravity sector. We show that Lorentz symmetry can be broken at much higher scales in a non-minimal theory with additional degrees of freedom, in particular if Lorentz symmetry is broken by the vacuum expectation value of a vector field. This theory can be constructed by gauging ghost condensation, giving a systematic effective field theory description that allows us to estimate the size of all physical effects. We show that nonlinear effects become important for gravitational fields with strength $\sqrt{\Phi} \gtrsim g$, where g is the gauge coupling, and we argue that the nonlinear dynamics is free from singularities. We then analyze the phenomenology of the model, including nonlinear dynamics and velocity-dependent effects. The strongest bounds on the gravitational sector come from either black hole accretion or direction-dependent gravitational forces, and imply that the scale of spontaneous Lorentz breaking is $M \lesssim \mathrm{Min}(10^{12}\ \mathrm{GeV}, g^2\, 10^{15}\ \mathrm{GeV})$. If the Lorentz breaking sector couples directly to matter, there is a spin-dependent inverse-square law force, which has a different angular dependence from the force mediated by the ghost condensate, providing a distinctive signature for this class of models.

1 Introduction

As the 100th anniversary of special relativity comes to a close, Lorentz invariance reigns as arguably the most important symmetry in modern physics. A good way to understand the central role of Lorentz invariance is to consider how it could be violated. Like baryon and lepton number in the standard model, Lorentz invariance could be an accidental symmetry of the leading interactions in quantum field theory. Alternatively, Lorentz symmetry could be one limit of a more fundamental symmetry, just as the Galilean group is the small velocity limit of the Lorentz group. But in the absence of experimental data, it is difficult to guess what deeper organizing principle could replace Lorentz invariance. Indeed, the Lorentz group itself is one of many deformations of the Galilean group, and it was the experimental crisis of the ether that drove the transition from absolute time to the space-time continuum in 1905.

Here, we consider the less radical possibility that Lorentz invariance is a good symmetry at high energies, but is spontaneously broken at an energy scale M. Some of the consequences of breaking Lorentz invariance have been extensively explored in the literature. If the symmetry breaking sector couples to standard model fields, there will be Lorentz-violating operators in the effective theory that give rise to CPT violation and a variety of preferred frame effects (see Ref. [1, 2, 3, 4, 5, 6, 7] and references therein). A second aspect that has been received less attention is that spontaneous breaking of Lorentz invariance implies the existence of new degrees of freedom analogous to the Goldstone bosons that arise from spontaneous breaking of internal symmetries.

The minimal model of this kind was analyzed in Ref. [8], which contains a single Goldstone degree of freedom. It can be viewed as the result of ghost condensation (in the same way that ordinary spontaneous symmetry breaking can be viewed as "tachyon condensation") and so we call the extra degree of freedom a "ghostone" boson. If the ghostone mode couples to the standard model, then it gives rise to exotic spin- and velocity-dependent forces [8, 9]. In a gravitational theory, breaking Lorentz invariance immediately implies a violation of diffeomorphisms, the gauge group of Einstein gravity, and thus the ghostone mode will mix with the metric. This is the Higgs mechanism for gravity, and is analogous to the familiar Higgs mechanism for gauge theories, where the mixing between a gauge field and a Goldstone boson gives rise to a massive spin-1 boson. Refs. [8, 10] analyzed the modification of gravity that arises in this way, and found the limit $M \lesssim 100$ GeV because of an infrared gravitational instability analogous to the Jeans instability for ordinary matter. Therefore, spontaneous Lorentz breaking is directly tied to modifications of gravity, and we can

place strong constraints on Lorentz breaking by looking at gravitational physics.

In this paper, we consider a non-minimal model of spontaneously broken Lorentz invariance that has three extra degrees of freedom instead of a single ghostone mode. We will show that in this model it is possible to break Lorentz invariance spontaneously at much higher scales, as high as $M \sim \mathrm{Min}(10^{12}\ \mathrm{GeV},\, g^2\, 10^{15}\ \mathrm{GeV})$. The model can be thought of as arising from a vector field order parameter, and has been previously considered in the literature [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. There are several features of our analysis that are new. First, we show that this model can be viewed as a gauging (in the ordinary sense) of ghost condensation. This considerably clarifies the nature of the effective theory, and allows us to consistently estimate the size of all operators in a low energy expansion. Second, we consider the couplings of the Goldstone degrees of freedom to matter, and show that there are new spin- and velocity-dependent forces with a different signature from the minimal model. Third, we show that the theory has important nonlinear corrections in strong gravitational fields. We find that the nonlinear effects are much less dramatic than in ghost condensation (see Ref. [10]), and argue that the theory is free from caustic singularities. Finally, we consider the modification of gravity in this model, studying observable consequences such as velocity-dependent effects (such as gravitational Čerenkov radiation), nonlinear dynamics, and the accretion of the condensate by black holes. Our conclusion is that the model is consistent with all experimental limits for $M \lesssim \mathrm{Min}(10^{12}\ \mathrm{GeV},\, g^2\, 10^{15}\ \mathrm{GeV})$, where g is the gauge coupling and other dimensionless coefficients are taken as $\mathcal{O}(1)$.

This paper is organized as follows. In Sec. 2, we describe the effective theory by directly introducing the extra degrees of freedom required by diffeomorphism invariance. This is the most direct route to the effective theory and the modification of gravity. In Sec. 3, we describe the same effective theory as the gauged version of ghost condensation. This formulation makes it manifest that this is a theory of *spontaneous* breaking of Lorentz symmetry, and also makes it easy to understand the general power counting and the construction of the nonlinear theory. In Sec. 4, we couple the gauged ghost condensate to gravity and perform a linear analysis. We find that the modification of gravity is mild in this case and it is the form of a direction-dependent Newtonian potential. In Sec. 5, we study the nonlinear effects. In contrast to the ungauged ghost condensate, the would-be caustics are smoothed out by the gauge interaction and there are no dangerous instabilities. In Sec. 6, we consider gauged ghost condensate surrounding a black hole and calculate the rate of increase of the black hole mass due to accretion of the condensate. In Sec. 7, we compute the energy loss due to the ether Čerenkov radiation for a source moving faster than

the ghostone mode. In Sec. 8, we study various Lorentz-violating effects and their constraints if the gauged ghost condensate couples directly to the Standard Model fields. Sec. 9 contains our conclusions. Some more detailed analyses and discussion are included in the appendices.

2 Lorentz Violation and Diffeomorphisms

In a non-gravitational theory, Lorentz symmetry can be broken explicitly without introducing any Goldstone degrees of freedom. The reason is that Lorentz symmetry is a physical symmetry that arranges matter states into multiplets, so violating Lorentz symmetry will only affect the allowed couplings between matter states.[1] As we will see, however, breaking Lorentz invariance implies a breaking of diffeomorphism invariance, the gauge symmetry of Einstein gravity. Since a gauge symmetry is a redundancy of description rather than a physical symmetry, breaking diffeomorphisms necessarily introduces additional degrees of freedom. The key point is that these new modes nonlinearly realize diffeomorphisms (as well as nonlinearly realizing Lorentz transformations), and therefore their couplings to gravity are uniquely determined by diffeomorphism invariance.

In this section, we show how to construct the effective theory below the Lorentz breaking scale by explicitly adding these degrees of freedom to the Lagrangian. We will also see that different ways of nonlinearly realizing diffeomorphisms give rise to different low energy effective theories. We review the case of ghost condensation which has only one new degree of freedom and introduce "gauged ghost condensation" which has three new degrees of freedom. We find that the mixing between gravity and the gauged ghost condensate is mild, which explains why the scale of spontaneous Lorentz breaking can be much higher in this theory. Other interesting generalizations are also possible, see for example Refs. [24, 25, 26].

The simplest way to break Lorentz invariance is to include the time component of a vector field in the Lagrangian:

$$\Delta\mathcal{L} = J^0. \tag{2.1}$$

The arguments below are easily generalized to components of higher tensors. Eq. (2.1) breaks Lorentz symmetry down to $SO(3)$ rotations, and also breaks diffeomorphism symmetry. Moreover, there is no way to couple $\Delta\mathcal{L}$ to the metric $g_{\mu\nu}$ to restore

[1] Of course, one can spontaneously break Lorentz symmetry in a non-gravitational theory, which would give rise to Goldstone modes. The point is that in the absence of gravity, Lorentz violation can be explicit, whereas in a gravitational theory, Lorentz violation must be spontaneous.

diffeomorphisms. To see this, we work in the linearized theory obtained by expanding the metric about flat space (where Lorentz invariance has an unambiguous meaning)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{2.2}$$

Under the infinitesimal diffeomorphism generated by $x^\mu \to x^\mu - \xi^\mu$, the metric and J^μ transform as

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{2.3}$$

$$\delta J^\mu = \xi^\rho \partial_\rho J^\mu - \partial_\rho \xi^\mu J^\rho. \tag{2.4}$$

We have

$$\delta(\sqrt{-g}\, J^0) = -\sqrt{-g}\, \partial_\mu \xi^0 J^\mu, \tag{2.5}$$

which makes it clear that there is no way to introduce factors of $h_{\mu\nu}$ to make $\Delta\mathcal{L}$ invariant under diffeomorphisms.

However, we can formally restore diffeomorphism invariance by introducing a scalar field π that transforms as

$$\delta\pi = \xi_0. \tag{2.6}$$

This is the degree of freedom in the "ghost condensate" model of Ref. [8], and π naturally has dimensions of length. The combination

$$\Delta\mathcal{L} = \sqrt{-g}\, (J^0 + \partial_\mu \pi J^\mu) \tag{2.7}$$

is fully diffeomorphism invariant.[2] Note that diffeomorphism invariance is realized nonlinearly, since the π field transforms inhomogeneously.

Because π is a real propagating degree of freedom, we need to introduce a kinetic term for π, but diffeomorphism invariance requires the kinetic term to depend on $h_{\mu\nu}$. Specifically, the time kinetic term

$$\mathcal{L}_{\pi\,\text{kinetic}} = +\tfrac{1}{2} M^4 (\dot{\pi} - \tfrac{1}{2} h_{00})^2 \tag{2.8}$$

is diffeomorphically invariant in the linearized theory. Note that the sign is fixed by the requirement that π fluctuations have positive energy. Eq. (2.8) shows that diffeomorphism invariance requires mixing between the new degree of freedom and gravity. In a gauge where $\pi \equiv 0$, Eq. (2.8) becomes a "wrong sign" mass term

[2] In fact, Eq. (2.7) is invariant in full nonlinear Einstein gravity.

for h_{00}, suggesting an oscillatory Newtonian potential at distances larger than the inverse "mass" $r_c \sim M_{\rm Pl}/M^2$. A more careful analysis shows that the potential is indeed modified at distances larger than r_c, but only on a much larger time scale $t_c \sim M_{\rm Pl}^2/M^3$ [8]. The theory is unstable on time scales of order t_c, and demanding that this is longer than the present age of the universe gives $M \lesssim 10$ MeV, though nonlinear effects slightly relax this bound to $M \lesssim 100$ GeV [10].

The strong bound on M arises because the way we have restored diffeomorphism invariance allows a graviton "mass".[3] To find a theory that allows larger values of M, we look for a way of restoring diffeomorphism invariance that does not necessitate a graviton mass term. To do this, note that

$$\delta[\sqrt{-g}\,(1 + \tfrac{1}{2}h_{00})J^0] = \sqrt{-g}\,\partial_i \xi^0 J^i, \tag{2.9}$$

where the sum over i does not include any additional minus signs. This shows that we can restore difformorphism invariance by introducing three fields a_i transforming as

$$\delta a_i = \partial_i \xi_0. \tag{2.10}$$

Note that a_i defined in this way is naturally dimensionless. Because these fields transform as the gradient of a diffeomorphism parameter, the time kinetic term involves mixing with derivatives of $h_{\mu\nu}$:

$$\mathcal{L}_{a\,\rm kinetic} \sim +M^2(\dot{a}_i - \tfrac{1}{2}\partial_i h_{00})^2. \tag{2.11}$$

It is easy to see that all diffeomorphism-invariant combinations of a_i and $h_{\mu\nu}$ involve derivatives acting on the metric, so modifications of Newton's law are suppressed. We will analyze this theory in more detail starting in Sec. 3, and we will see that this construction allows much larger values for the scale M where Lorentz symmetry is broken.

To see the connection between the above theory and gauging ghost condensation, note that the diffeomorphism transformation Eq. (2.10) is very similar to an ordinary $U(1)$ gauge transformation

$$\delta a_\mu = \partial_\mu \lambda. \tag{2.12}$$

In fact, we can promote Eq. (2.10) to an ordinary gauge transformation by introducing a field π that transforms under gauge transformations and diffeomorphisms as

$$\delta\pi = \xi_0 - \lambda. \tag{2.13}$$

[3]Strictly speaking, only the Newtonian potential gets a mass; the propagating spin-2 gravitons are still massless.

The combination

$$\tilde{a}_\mu = a_\mu + \partial_\mu \pi \tag{2.14}$$

is then gauge invariant and transforms under diffeomorphisms as

$$\delta \tilde{a}_\mu = \partial_\mu \xi_0. \tag{2.15}$$

In "unitary gauge" $\pi \equiv 0$ we have $\tilde{a}_i = a_i$, so the spatial components can be identified with the fields introduced in equation Eq. (2.10). The extra degree of freedom $\tilde{a}_0$ is naturally massive, since nothing forbids the term

$$\mathcal{L}_{a_0\,\mathrm{mass}} = +\tfrac{1}{2} M^4 (\tilde{a}_0 - \tfrac{1}{2} h_{00})^2 \tag{2.16}$$

which is a mass term for a_0 in unitary gauge.

As we will explain in sections 3 and 4, Eq. (2.16) reduces to Eq. (2.8) when we take the a_μ gauge coupling to zero, which explains why the modification of gravity is suppressed in gauged ghost condensation. We find that for canonically normalized fields in unitary gauge, Eq. (2.16) becomes

$$\mathcal{L}_{\mathrm{mass}} = \frac{M^2}{2} \left(g A_0^c - g_c \Phi^c \right)^2, \qquad g_c = \frac{M}{\sqrt{2} M_{\mathrm{Pl}}}, \tag{2.17}$$

where A_μ^c is the canonically normalized gauge field, Φ^c is the canonically normalized Newtonian potential, and g is the A_μ^c gauge coupling. When g is smaller than g_c, the (negative) mass term dominantly affects the Newtonian potential, yielding large modifications to gravitational potentials as in ghost condensation. When g is larger than g_c, the mass term acts mostly on the "Coulomb" potential A_0^c, shielding Lorentz breaking from the gravitational sector and allowing the scale M in gauged ghost condensation to be raised as high as $M \lesssim \mathrm{Min}(10^{12}\ \mathrm{GeV},\ g^2\, 10^{15}\ \mathrm{GeV})$.

A discussion of the decoupling limit of gauged ghost condensation which makes contact with earlier literature on Lorentz breaking appears in App. A.

3 Gauged Ghost Condensation

In this section, we will start from the ghost condensate and construct the effective theory by gauging a global symmetry. This will clarify the power counting, and also show how the theory reduces to the ghost condensate in a particular limit.

3.1 A Review of Ghost Condensation

We first review the "ghost condensate" model of Ref. [8]. This is an effective theory of a real scalar field ϕ with a global shift symmetry

$$\phi \to \phi + \lambda. \tag{3.1}$$

The effective theory is assumed to depend on a single scale M, which gives both the strength of self-interactions of the ϕ field and the cutoff of the effective theory. The novel feature of this sector is that it has a vacuum (in flat spacetime)

$$\langle \partial_\mu \phi \rangle = v_\mu, \tag{3.2}$$

where v_μ is a constant timelike vector. This spontaneously breaks Lorentz invariance by establishing a preferred time direction. We can then choose a Lorentz frame where $v_0 = c$, $v_i = 0$ for $i = 1, 2, 3$. In this frame we have

$$\langle \phi \rangle = ct. \tag{3.3}$$

It is convenient to take ϕ to have units of time, so that c is dimensionless and can be set to 1 by rescaling the time coordinate.

Expanding about the vacuum

$$\phi = \langle \phi \rangle + \pi, \tag{3.4}$$

we can construct a systematic effective field theory for the fluctuations π in derivatives simply by noting that

$$\begin{aligned} \partial_\mu \phi &= \delta_\mu{}^0 + \partial_\mu \pi, \\ \partial_\mu \partial_\nu \phi &= \partial_\mu \partial_\nu \pi. \end{aligned} \tag{3.5}$$

The constant part of $\partial_\mu \phi$ should be kept to all orders since there is no momentum suppression. On the other hand, terms with more than one derivative acting on ϕ only give rise to derivatives acting on the fluctuations, so there is a well-defined expansion at low energies and only the lowest order terms are important. The most general quadratic effective Lagrangian with at most four derivatives acting on π is

$$\begin{aligned} \mathcal{L}_{\text{ghost}} = M^4 P(X) + \tfrac{1}{2} M^2 \Big(& Q_1(X)(\Box \phi)^2 + Q_2(X) \partial^\mu \phi \partial^\nu \phi \partial_\mu \partial_\nu \phi \Box \phi \\ & + Q_3(X)(\partial^\mu \phi \partial^\nu \phi \partial_\mu \partial_\nu \phi)^2 \Big) + \cdots, \end{aligned} \tag{3.6}$$

where we have assumed a $\phi \to -\phi$ reflection symmetry, and

$$X = \partial^\mu \phi \partial_\mu \phi. \tag{3.7}$$

Expanding this Lagrangian about the vacuum using Eq. (3.5), the leading Lagrangian is

$$\mathcal{L}_{\text{ghost}} = \tfrac{1}{2} M^4 \dot{\pi}^2 - \tfrac{1}{2} M^2 \Big(\alpha (\vec{\nabla}^2 \pi)^2 + \beta \ddot{\pi} \vec{\nabla}^2 \pi - \gamma \ddot{\pi}^2 \Big) + \mathcal{O}\left(\pi^3, (\partial^3 \pi)^2 \right). \tag{3.8}$$

Here we have rescaled the fields so that $P''(1) = \frac{1}{4}$, which fixes the coefficient of the $\dot{\pi}^2$ term, and used the fact that $P'(1) = 0$ in the vacuum so that there is no tadpole term. The coefficients of the other terms are given by

$$\begin{aligned} \alpha &= -Q_1(1), \\ \beta &= 2Q_1(1) + Q_2(1), \\ \gamma &= Q_1(1) + Q_2(1) + Q_3(1). \end{aligned} \tag{3.9}$$

We can further rescale π and M to simplify this Lagrangian, but we will not make use of this freedom for now. We can see that there is no normal spatial kinetic term $(\vec{\nabla}\pi)^2$, which implies the special dispersion relation for π

$$\omega^2 = \alpha \frac{\vec{k}^4}{M^2} \tag{3.10}$$

to leading order in ω and k.

3.2 Gauging Ghost Condensation

We now couple the theory above to a gauge field A_μ by gauging the shift symmetry in Eq. (3.1). That is, we promote Eq. (3.1) to a local transformation and introduce a gauge field to make the Lagrangian invariant. It is convenient to define the gauge field as $A_\mu = M a_\mu$ so that the gauge field has dimensions of mass. We then write the gauge transformations as

$$\delta A_\mu = \partial_\mu \chi, \tag{3.11}$$

where χ is dimensionless. The gauge transformation of ϕ is then

$$\delta \phi = -\frac{1}{M} \chi. \tag{3.12}$$

We have rescaled χ and A_μ to fix the coefficients in these transformation rules. The fields A_μ and ϕ must always appear in the gauge-invariant combination

$$\mathcal{A}_\mu = A_\mu + M\partial_\mu\phi, \tag{3.13}$$

which can also be thought of as the gauge covariant derivative of ϕ: $\mathcal{A}_\mu = MD_\mu\phi$.

We assume that the theory violates Lorentz invariance in the vacuum via the gauge-invariant order parameter

$$\langle \mathcal{A}_\mu \rangle = M v_\mu, \tag{3.14}$$

where v_μ is a timelike vector of unit norm. (This defines the scale M.) In the preferred frame, the order parameter is $\langle \mathcal{A}_0 \rangle$.

The coupling of the gauge field to the ghost condensate is governed by gauge invariance. Starting from Eq. (3.8), we can write terms

$$\mathcal{L}_{\text{gauged ghost}} = -\frac{1}{4g^2} F^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{\text{ghost}}(\partial_\mu\phi \to \mathcal{A}_\mu/M). \tag{3.15}$$

This does not uniquely fix the leading terms, since we can obtain gauge invariant terms by replacing $\partial_\mu\partial_\nu\phi$ with either $\partial_\mu\mathcal{A}_\nu$ or $\partial_\nu\mathcal{A}_\mu$. The general linearized Lagrangian expanded about the vacuum $\langle \mathcal{A}_\mu \rangle = M\delta_\mu{}^0$ is then

$$\begin{aligned}\mathcal{L}_{\text{gauged ghost}} = &-\frac{1}{4g^2} F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}M^2(\mathcal{A}_0 - M)^2 \\ &- \tfrac{1}{2}\alpha_1(\partial_i\mathcal{A}_i)^2 - \tfrac{1}{2}\alpha_2(\partial_i\mathcal{A}_j)^2 + \tfrac{1}{2}\beta_1(\partial_t\mathcal{A}_i)^2 - \tfrac{1}{2}\beta_2(\partial_i\mathcal{A}_0)^2 \\ &+ \beta_3\partial_t\mathcal{A}_0\partial_i\mathcal{A}_i + \tfrac{1}{2}\gamma(\partial_t\mathcal{A}_0)^2 + \mathcal{O}(\partial^2\mathcal{A}^4) + \mathcal{O}(\mathcal{A}^3).\end{aligned} \tag{3.16}$$

By the power counting of the ghost condensate effective theory, the coefficients $\alpha_i, \beta_i, \gamma$ are order 1, and give rise to general kinetic terms for the vector fields. While the $F^{\mu\nu}F_{\mu\nu}$ term does not generate any independent gauge kinetic terms, for $g \ll 1$ it gives the dominant kinetic contribution to the transverse modes. In the limit $g \to 0$, we recover the ghost condensate theory.

Eq. (3.16) contains a "mass" term for $\mathcal{A}_0$, so we can integrate out $\mathcal{A}_0$ if we are interested in energies and momenta below the scale M. To see this explicitly, we compute the dispersion relation for the scalar modes. We parametrize them by

$$A_0 = M + a, \qquad A_i = \partial_i\sigma, \qquad \pi. \tag{3.17}$$

Choosing "unitary gauge" $\pi \equiv 0$, the quadratic Lagrangian in momentum space is

$$\mathcal{L} = \frac{1}{2g^2} \begin{pmatrix} \sigma & a \end{pmatrix} K \begin{pmatrix} \sigma \\ a \end{pmatrix}, \tag{3.18}$$

where

$$K = \begin{pmatrix} (1+g^2\beta_1)\omega^2\vec{k}^2 - g^2\alpha\vec{k}^4 & i(1-g^2\beta_3)\omega\vec{k}^2 \\ -i(1-g^2\beta_3)\omega\vec{k}^2 & (1-g^2\beta_2)\vec{k}^2 + g^2\gamma\omega^2 + g^2M^2 \end{pmatrix}, \qquad (3.19)$$

and $\alpha \equiv \alpha_1 + \alpha_2$. The dispersion relation for the scalar modes is obtained by setting the determinant of the kinetic matrix to zero. We find two scalar modes, one with dispersion relation

$$\omega^2 = \frac{\alpha g^2}{1+g^2\beta_1}\vec{k}^2 + \left(\frac{1-g^2\beta_3}{1+g^2\beta_1}\right)^2 \frac{\alpha\vec{k}^4}{M^2} + \mathcal{O}(\vec{k}^6/M^2), \qquad (3.20)$$

and one with

$$\omega^2 = -\frac{M^2}{\gamma} + \mathcal{O}(\vec{k}^2). \qquad (3.21)$$

The first mode is gapless and corresponds roughly to the longitudinal mode σ. For sufficiently small $\vec{k}$ we can neglect the quartic term in the dispersion relation, so the gapless modes travel with constant speed in the preferred frame. For $g \ll 1$, modes with $|\vec{k}| \ll gM$ have speed

$$c_s = \sqrt{\alpha}\, g \ll 1. \qquad (3.22)$$

Gapless modes with $|\vec{k}| \gg gM$ have a quartic dispersion relation, just as the Goldstone boson π in the (ungauged) ghost condensate. In this way, this theory approaches the theory of Ref. [8] as $g \to 0$. In the opposite limit $g \gg 1$ the speed is $\mathcal{O}(1)$ for the gapless mode with $|\vec{k}| \ll M$, so the quartic term is never important within the regime of validity of the effective theory.

The second mode has an energy gap of order M, and is therefore not a mode that is accurately described in the effective theory. We therefore write the effective theory below the scale M by integrating out the field A_0, which has an order-1 overlap with the massive mode. We can do this without fixing any gauge, and we obtain the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{2g_t^2}(\partial_t\vec{\mathcal{A}})^2 - \frac{1}{4g_s^2}F_{ij}^2 - \tfrac{1}{2}\alpha(\vec{\nabla}\cdot\vec{\mathcal{A}})^2 + \mathcal{O}(1/M^2), \qquad (3.23)$$

where

$$\frac{1}{g_t^2} = \frac{1}{g^2} + \beta_1, \qquad \frac{1}{g_s^2} = \frac{1}{g^2} + \alpha_2. \qquad (3.24)$$

We have dropped $1/M^2$ terms in Eq. (3.23), which means that we have neglected the $\vec{k}^4$ term in the dispersion relation for the massless scalar mode. For $g \ll 1$, this is only justified if we restrict attention to momenta satisfying $|\vec{k}| \ll gM$.[4]

What has happened is that the Goldstone mode π has been "eaten" by the gauge field A_μ and can be viewed as the longitudinal mode of the gauge field $\mathcal{A}_\mu$. However, unlike in the Lorentz-invariant Higgs mechanism, only $\mathcal{A}_0$ gets a mass term and the modes parametrized by $\mathcal{A}_i$ remain massless. In a sense, the Lorentz-violating Higgs mechanism shields "electric" interactions ($\mathcal{A}_0$) while preserving "magnetic" ones ($\mathcal{A}_i$). We will explore this interesting feature when we consider coupling the gauged ghost condensate to the standard model in Sec. 8.

4 Coupling the Gauged Ghost Condensate to Gravity

We now analyze the coupling of the gauged ghost condensate to gravity. We will see that the modification of gravity is much less dramatic than in ghost condensation. One aspect of this was already discussed in Sec. 3, namely that the vector modes mix with gravity only via derivative terms, and therefore do not modify the static Newtonian potential, at least in the preferred frame. Another aspect comes from the result of the previous section, that the gauged ghost condensate has a dispersion relation $\omega^2 \simeq c_s \vec{k}^2$ for small $\vec{k}$, so long wavelength fluctuations are stable. Coupling to gravity can give only additional terms suppressed by $1/M_{\rm Pl}^2$, so we do not expect any instabilities in the presence of gravity. This is in contrast to ghost condensation, where the theory without gravity has a dispersion relation $\omega^2 \sim \vec{k}^4/M^2$, and gravity gives a correction $\Delta\omega^2 \sim -M^2\vec{k}^2/M_{\rm Pl}^2$ that generates an instability at long wavelengths.

Since we start with a completely generally covariant description of the theory, the coupling to gravity is determined simply by promoting the metric to a dynamical degree of freedom and replacing spacetime derivatives ∂_μ by covariant derivatives ∇_μ. For the full nonlinear theory, this is

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm EH} + \sqrt{-g}\Big[&-\frac{1}{4g^2} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} + \tfrac{1}{4} M^2 (g^{\mu\nu}\mathcal{A}_\mu \mathcal{A}_\nu - M^2)^2 \\ &+ \tfrac{1}{2}\alpha_1 (g^{\mu\nu}\nabla_\mu \mathcal{A}_\nu)^2 + \cdots \Big].\end{aligned} \tag{4.1}$$

[4] The $\vec{k}^4$ correction can be obtained by keeping the term

$$\Delta\mathcal{L} = -\frac{1}{2g^4 M^2}(\partial_t \partial_i \mathcal{A}_i)^2,$$

which is enhanced for small g.

where $\mathcal{L}_{\rm EH}$ is the Einstein-Hilbert action.

4.1 Linear Theory

Expanding about flat spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{4.2}$$

the quadratic terms in the Lagrangian are

$$\mathcal{L} = \mathcal{L}_{\rm EH} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \tfrac{1}{2} M^2 (\mathcal{A}_0 - \tfrac{1}{2} M h_{00} - M)^2 + \cdots \tag{4.3}$$

The weak gauging of gravity affects the dispersion relation for the scalar modes only by terms suppressed by $1/M_{\rm Pl}$, so we can still integrate out the massive mode A_0 as before. The leading quadratic terms in the effective Lagrangian for small g is then

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm EH} + \frac{1}{2g_t^2} (\nabla_t\, \mathcal{A}_i)^2 - \frac{1}{4g_s^2} F_{ij}^2 - \tfrac{1}{2} \alpha (\partial_i \mathcal{A}_i)^2 + \cdots. \tag{4.4}$$

Here

$$\nabla_t\, \mathcal{A}_i = \partial_t \mathcal{A}_i - \frac{M}{\sqrt{2} M_{\rm Pl}} \partial_i \Phi^c, \tag{4.5}$$

where $\Phi^c = \sqrt{2} M_{\rm Pl} \Phi = M_{\rm Pl} h_{00}/\sqrt{2}$ is the canonically normalized Newtonian potential. Note that the $(\nabla_t\, \mathcal{A}_i)^2$ term contains mixing between $\mathcal{A}_i$ and h_{00}.

The effective Lagrangian in Eq. (4.4) is fully invariant under gauge transformations and diffeomorphisms:

$$\begin{aligned} \delta\pi &= -\xi_0 - \frac{\chi}{M}, \\ \delta A_i &= \partial_i \chi, \\ \delta h_{\mu\nu} &= -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu. \end{aligned} \tag{4.6}$$

In fact, the quadratic terms in Eq. (4.4) are the most general ones invariant under these symmetries, and armed with the foresight to integrate out A_0, we could have used Eq. (4.6) as the starting point for our analysis. Indeed, this is exactly what we suggested in Sec. 2 when we posited the existence of a a_i field. One advantage of thinking of Eq. (4.4) as coming from gauged ghost condensation is that the weak gauging procedure explains why g_t and g_s are nearly degenerate.

The modified dispersion relation for the scalar excitation can be easily derived. Parameterizing $\mathcal{A}_i = \partial_i \sigma$ and choosing Newtonian gauge, we have

$$\mathcal{L} = \frac{1}{2} \begin{pmatrix} \sigma/g & \Phi^c \end{pmatrix} \begin{pmatrix} \omega^2 \vec{k}^2 - \alpha g^2 \vec{k}^4 & -i\epsilon\omega\vec{k}^2 \\ i\epsilon\omega\vec{k}^2 & -\vec{k}^2(1-\epsilon^2) \end{pmatrix} \begin{pmatrix} \sigma/g \\ \Phi^c \end{pmatrix} + \cdots, \tag{4.7}$$

where

$$\epsilon = \frac{M}{\sqrt{2}\, g M_{\rm Pl}}, \tag{4.8}$$

and we have taken the small g limit and dropped the subleading terms suppressed by g^2.[5] Setting the determinant of the kinetic matrix to zero, we obtain

$$\omega^2 = c_s^2\, \vec{k}^2, \tag{4.9}$$

where

$$c_s^2 = \alpha(g^2 - g_c^2), \qquad g_c = \epsilon g = \frac{M}{\sqrt{2} M_{\rm Pl}}. \tag{4.10}$$

We can see that if $g > g_c$, this is a healthy dispersion relation and there is no instability even at arbitrarily long wavelengths. If we also keep the leading contribution at $\mathcal{O}(\vec{k}^4/M^2)$, then the dispersion relation is

$$\omega^2 = c_s^2\, \vec{k}^2 + \alpha \frac{\vec{k}^4}{M^2}, \tag{4.11}$$

which reproduces the behavior of ghost condensation for $|\vec{k}| \gg gM$.

The dispersion relation of the tensor modes is also modified because $(\nabla_i \mathcal{A}_j)^2$ contains a term $\frac{1}{2}\langle \mathcal{A}_0 \rangle \partial_t h_{ij}$, which contributes to the time derivative of h_{ij}. As a result, the speed of usual gravitational waves is modified by $\mathcal{O}(\alpha_2 M^2/M_{\rm Pl}^2)$. If the speed of the gravitational wave is smaller than the speeds of ordinary particles, there is a strong constraint from cosmic rays [27, 28].

4.2 Modification of Gravity and Observational Constraints

Because of the mixing between the Goldstone boson and the metric tensor, the gravitational potential is modified. In the static limit, $\omega \to 0$, the only modification is a simple renormalization of Newton's constant,

$$G_N = \frac{G_N^0}{1-\epsilon^2} = \frac{1}{8\pi M_{\rm Pl}^2 (1-\epsilon^2)}, \tag{4.12}$$

[5] The second term in the $(1,1)$ entry gives the leading $\vec{k}^2$ term in the dispersion relation and therefore must be kept.

as one can see directly from Eq. (4.7). By itself, this does not lead to any measurable effects.

There are non-trivial effects if the source is moving relative to the ether rest frame, as expected in realistic situations. For a source of mass $M_\odot$ moving with velocity $\vec{v}$ relative to the ether rest frame, the stress-energy tensor is

$$T_{00} = \rho = M_\odot \delta^{(3)}(\vec{x} - \vec{v}t), \tag{4.13}$$

and T_{0i} and T_{ij} are down by powers of $v = |\vec{v}|$ and can be ignored for $v \ll 1$. The gravitational potential can be easily obtained through the $\Phi\Phi$ propagator by inverting the kinetic matrix in Eq. (4.7),

$$\langle \Phi\Phi \rangle = -\frac{1}{2M_{\rm Pl}^2}\frac{1}{\vec{k}^2}\left(1 - \frac{\alpha g^2 \epsilon^2 \vec{k}^2}{\omega^2 - \alpha g^2(1-\epsilon^2)\vec{k}^2}\right). \tag{4.14}$$

The Fourier transform of the source is

$$\tilde{\rho}(\omega, \vec{k}) = 2\pi M_\odot \delta(\omega - \vec{k}\cdot\vec{v}). \tag{4.15}$$

The Newtonian potential is obtained by performing the inverse Fourier transform of the $\langle\Phi\Phi\rangle$ propagator after substituting ω by $\vec{k}\cdot\vec{v}$. Depending on the source velocity in the preferred frame v relative to the velocity of the scalar Goldstone mode $c_s \equiv \sqrt{\alpha(g^2 - g_c^2)}$, we get very different results. It is important to distinguish the two different cases

For $v < c_s$, we can expand in powers of v/c_s and obtain the angular-dependent gravitational potential

$$V(r,\theta) = -\frac{G_N^0 M_\odot}{r}\left[1 + \frac{\epsilon^2}{1-\epsilon^2}\left(1 + \frac{v^2}{2c_s^2}\sin^2\theta + \mathcal{O}(v^4/c_s^4)\right)\right], \tag{4.16}$$

where $\cos\theta = \hat{r}\cdot\hat{v}$ is the angle measured from the direction of the source moving with respect to the ether. In the parametrized post-Newtonian (PPN) formalism [31] this velocity effect with respect to the preferred frame corresponds to the PPN parameter $\alpha_2^{\rm PPN}$,

$$\alpha_2^{\rm PPN} = \frac{\epsilon^2}{(1-\epsilon^2)c_s^2} = \frac{M^2}{2\alpha g^4(1-\epsilon^2)^2 M_{\rm Pl}^2}. \tag{4.17}$$

The observational bound on $\alpha_2^{\rm PPN}$ is

$$\alpha_2^{\rm PPN} < 4\times 10^{-7} \tag{4.18}$$

from the alignment of the solar spin axis and its ecliptic [29, 30, 31]. This provides the strongest bound on the scale of Lorentz symmetry breaking in this case.[6] For αg^4 close to 1 we have $M \lesssim 10^{15}$ GeV, and the constraint is stronger ($M \lesssim 10^{15}\sqrt{\alpha} g^2$ GeV) for smaller g. This bound was also considered in Ref. [33] using the parametrization of Ref. [14] (which corresponds to a decoupling limit of our theory as shown in App. A). A more systematic derivation of the modification of gravity and discussion of other PPN parameters can be found in Appendices B and C. In particular, we find that $\alpha_2^{\rm PPN}$ is the only modification of General Relativity at post-Newtonian order, showing that gauged ghost condensation yielding a very mild modification of gravity. Our analysis also exhibits the applicable range of the PPN formalism in this theory. *It is not enough just for v to be small because the expansion parameter is actually v/c_s.* It requires $v < c_s$ and the mixing parameter ϵ to be small. The above result holds for any theory where Lorentz invariance is broken by the vev of a vector field (such as Refs. [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]).

On the other hand, if $v > c_s$, the PPN expansion breaks down. We expect that there will be Čerenkov radiation into the Goldstone field. By causality, modifications of gravity happen only in a cone with an angle $\theta = \sin^{-1}(c_s/v)$ behind the source.[7] For $v \gg c_s$, the cone is narrow and we may not see modifications of gravity in the solar system where the most stringent bounds come from, if the cone lies outside the ecliptic plane. (Of course, there will be some anomalous acceleration if an astrophysical object happens to move into the cone of shadow of a gravitational source.) The most interesting effects in this case probably come from the energy loss due to the Čerenkov radiation. Because the coupling of the Goldstone mode to gravity is proportional to M, the Čerenkov radiation due to gravity scales as M^2. The ability to raise the Lorentz symmetry breaking scale M in gauged ghost condensation makes this effect interesting. In fact, it provides a significant bound on M. In contrast, in ungauged ghost condensation, the bound $M \lesssim 100$ GeV renders this effect totally irrelevant. However, to discuss the Čerenkov radiation from moving stars or planets, we needs to first understand the nonlinear effects and where they become important, so we postpone the discussion of Čerenkov radiation to Sec. 7 after we discuss nonlinear effects.

[6] There is also a bound from the rate of the cosmic expansion during the Big Bang nucleosynthesis due to the different effective Newton's constant for the cosmological evolution [32], but it is much weaker.

[7] This is true for distances larger than $(gM)^{-1}$. For $gM < |\vec{k}| < M$, the $\vec{k}^4$ term in the dispersion relation becomes more important than the $\vec{k}^2$ term, and the scalar velocity is enhanced and becomes $\vec{k}$ dependent, $c_s \sim \sqrt{\alpha}|\vec{k}|/M$. The distance scale where this is relevant is very small if M is high and g is not too small.

5 Nonlinear Effects

Up to now we have confined our analysis to the linear order in the fields. In this section, we show that nonlinear effects can be important for sufficiently strong gravitational fields, and analyze the dynamics in the nonlinear regime both analytically and numerically. We show that the nonlinear dynamics has a simple physical interpretation as the dynamics of a charged fluid. We use this picture to give a qualitative understanding of the nonlinear dynamics. In particular, we argue that the caustic singularities found in ungauged ghost condensation are not present in the gauged case.

5.1 The Nonlinear Regime

Recall that in ungauged ghost condensation, nonlinear effects are important for all interesting gravitational sources, such as stars and galaxies [10, 34]. The physical reason for this is that the ghost condensate gravitates like a fluid that obeys the equivalence principle, and therefore has a gravitational response time of order

$$t_{\text{grav}} \sim \frac{r}{\sqrt{\Phi}}. \tag{5.1}$$

where r is the distance to the gravitating source. This effect appears only at nonlinear order; at linear order, the ghost condensate has a highly suppressed response time due to the $\omega^2 = \alpha \vec{k}^4/M^4$ dispersion relation:

$$t_{\text{linear}} \sim \frac{M r^2}{\sqrt{\alpha}}. \tag{5.2}$$

When $t_{\text{grav}} \lesssim t_{\text{linear}}$, the nonlinear effects dominate. These nonlinear effects were studied analytically and numerically in Ref. [10], and it was shown that regions with $X - 1 < 0$ generally shrink to small size, resulting in a breakdown of the effective theory. This is not necessarily a disaster for the theory, since the energy involved in these singular regions is very small, but it is unfortunate that this behavior cannot be understood in the effective theory.

In the case of gauged ghost condensation, we expect that it also gravitates on a timescale t_{grav}, but the linear timescale is given by the more conventional dispersion relation $\omega^2 = c_s^2 \vec{k}^2$:

$$t_{\text{linear}} \sim \frac{r}{c_s}. \tag{5.3}$$

We therefore expect that gravitational effects become important when $t_{\rm grav} \lesssim t_{\rm linear}$, or

$$\Phi \gtrsim c_s^2. \tag{5.4}$$

This means that nonlinear effects become important only for sufficiently strong gravitational fields. This will be important in subsequent sections when we discuss bounds on the gauged ghost condensate.

The result Eq. (5.4) can be derived as follows. Expanding $X - 1$ to include the leading nonlinear interaction,

$$\tfrac{1}{2}(X-1) = \frac{\mathcal{A}_0}{M} - \Phi - \frac{1}{2M^2}\vec{\mathcal{A}}^{\,2} + \cdots, \tag{5.5}$$

the leading terms in the effective Lagrangian are

$$\mathcal{L} = \frac{M^2}{2}\left(\mathcal{A}_0 - M\Phi - \frac{1}{2M}\vec{\mathcal{A}}^{\,2}\right)^2 - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{2}(\vec{\nabla}\cdot\vec{\mathcal{A}})^2. \tag{5.6}$$

Integrating out the massive $\mathcal{A}_0$ mode,

$$\mathcal{A}_0 = M\Phi + \frac{1}{2M}\vec{\mathcal{A}}^2, \tag{5.7}$$

we obtain

$$\frac{1}{2g^2}F_{0i}^2 = \frac{1}{2g^2}E_i^2, \tag{5.8}$$

where we define the "electric field"

$$\vec{E} = \partial_t\vec{\mathcal{A}} - M\vec{\nabla}\Phi - \frac{\vec{\nabla}(\vec{\mathcal{A}}^{\,2})}{2M}. \tag{5.9}$$

The leading terms in the nonlinear effective Lagrangian are therefore

$$\mathcal{L} = \frac{1}{2g^2}E_i^2 - \frac{1}{4g^2}F_{ij}^2 - \frac{\alpha}{2}(\vec{\nabla}\cdot\vec{\mathcal{A}}). \tag{5.10}$$

The leading nonlinear effects are contained in the $\vec{\nabla}(\vec{\mathcal{A}}^{\,2})$ term in $\vec{E}$.

Nonlinear effects become important when the $\vec{\nabla}(\vec{\mathcal{A}}^{\,2})$ term becomes comparable to the time derivative term:

$$t_{\rm nonlinear} \sim \frac{Mr}{\vec{\mathcal{A}}}. \tag{5.11}$$

The field amplitude $\vec{\mathcal{A}}$ induced by a gravitational field Φ is given by

$$\vec{\mathcal{A}} \sim M\sqrt{\Phi}, \tag{5.12}$$

which gives

$$t_{\rm nonlinear} \sim \frac{r}{\sqrt{\Phi}} \sim t_{\rm infall}, \tag{5.13}$$

as anticipated.

We now show that the nonlinear dynamics of the effective Lagrangian Eq. (5.10) has a natural fluid interpretation, similarly to the one found for ungauged ghost condensation in Ref. [10]. This will be very useful in understanding the nonlinear dynamics of the theory.

We begin by noting that in the relativistic formulation, the field $\mathcal{A}_\mu$ defines the local preferred rest frame. In the effective theory with $\mathcal{A}_0$ integrated out, the theory naturally defines a fluid with a local 3-velocity given by

$$v_i = -\frac{\mathcal{A}_i}{M}. \tag{5.14}$$

The definition of $\vec{E}$ Eq. (5.9) can then be written as

$$\frac{Dv_i}{Dt} = -\partial_i \Phi + \frac{1}{M}\left(-E_i + F_{ij} v_j\right), \tag{5.15}$$

where

$$\frac{D}{Dt} = \partial_t + \vec{v} \cdot \vec{\nabla} \tag{5.16}$$

is the time derivative along the worldline of a fluid particle (also called the convective or Lagrangian derivative). Eq. (5.15) has the form of Newton's law for a fluid particle, with gravitational, electric, and magnetic forces on the right-hand side.

The equations for the electric and magnetic fields arise from the $\mathcal{A}_i$ equation of motion of the effective Lagrangian, and can be written as

$$\partial_t E_i + \partial_j F_{ij} = -v_i \partial_j E_j - \alpha g^2 M \partial_i \partial_j v_j. \tag{5.17}$$

This has the form of Ampére's law with an unconventional current density on the right-hand side. Another difference from conventional electrodynamics is the absence of Gauss' law. This is expected, since the Higgs mechanism implies that all 3 components of $\mathcal{A}_i$ represent physical degrees of freedom.

As discussed in the previous subsection, for sufficiently strong gravity we can neglect the αg^2 term in Eq. (5.17), at least as long as spatial gradients are not too large. In this case, there is a simple solution with $E_i = 0$, $F_{ij} = 0$ in which the fluid particles follow gravitational geodesics. This is the relevant solution in the case where the fluid is initially "at rest" in the presence of a gravitating source. Just as in the ungauged case, the fluid particles will "fall in" toward the source, giving a very direct and physical picture of why the gravitational time scale is relevant for the

nonlinear dynamics. The subsequent evolution of the fluid will in general give rise to caustic singularities. Near these caustic singularities, the higher-derivative αg^2 is important, and may resolve the singularity. In the fluid picture, the "electric" and "magnetic" forces are becoming important near the would-be caustic, and may cause the incoming fluid particles to "bounce." This is the question we address next.

5.3 Would-be Caustics

To address the question of caustic singularities in the nonlinear dynamics, we restrict attention to situations with spherical, cylindrical, or planar symmetry. We can treat these simultaneously by using the spatial metric

$$ds^2 = dr^2 + r^2 d\Omega_s^2 + \sum_{p=1}^{2-s} dx_p^2, \tag{5.18}$$

where

$$d\Omega_s^2 = \begin{cases} 0 & s = 0 \text{ (planar symmetry)} \\ d\theta^2 & s = 1 \text{ (cylindrical symmetry)} \\ d\theta^2 + \sin^2\theta d\varphi^2 & s = 2 \text{ (spherical symmetry)} \end{cases} \tag{5.19}$$

Since only $\mathcal{A}_r$ is nonzero, we have $F_{ij} = 0$, and "Newton's law" simplifies to

$$\frac{Dv_r}{Dt} = -\partial_r \Phi - \frac{1}{M} E_r, \tag{5.20}$$

where $D/Dt = \partial_0 + v_r \partial_r$. "Ampére's law" Eq. (5.17) can then be written in the form

$$\frac{D}{Dt}(r^s E_r) = -r^s j_r, \tag{5.21}$$

where

$$j_r = \alpha g^2 M \partial_r \left[\frac{1}{r^s} \partial_r (r^s v_r) \right]. \tag{5.22}$$

Eq. (5.21) says that the comoving "flux" $r^s E_r$ is changing according to the "current density" j_r. In particular, when j_r is negligible, the "flux" is conserved, a fact that is useful in interpreting the numerical results below.

We now consider an initial condition that can lead to a caustic singularity. That is, we assume that initially $v_r < 0$ near $r = 0$, corresponding to a situation where the fluid particles are heading toward the origin. More precisely, we assume that near $r = 0$, we have

$$v_r = -c_1 r + c_2 r^2 + \mathcal{O}(r^3), \tag{5.23}$$

with $c_1 > 0$. Because we expect the linear growth of v_r away from $r = 0$ to degrease with r, we also assume $c_2 > 0$. In this case, we find that near $r = 0$

$$j_r = (s+2)\alpha g^2 M c_2 > 0. \tag{5.24}$$

Therefore, Eq. (5.21) implies that the "electric field" E_r tends to *decrease* with time. In particular, if $E_r = 0$ initially (*e.g.* if the fluid is "at rest") then later $E_r < 0$.

The sign of E_r is crucial for the fate of the would-be caustic. For $E_r < 0$, the "electric" force in Eq. (5.20) is repulsive and can cause the incoming particles to bounce. To see the condition for this, we rewrite Eq. (5.20) as a function of the comoving radial coordinate r as

$$\frac{D}{Dr}\left(\tfrac{1}{2}v^2\right) = -\frac{1}{M}E_r, \tag{5.25}$$

where we have neglected the gravitational "force." We therefore have

$$\Delta\left(\tfrac{1}{2}v^2\right) = -\frac{1}{M}\int_{r_i}^{r_f} dr\, E_r. \tag{5.26}$$

This can be interpreted as the work-energy theorem for fluid particles. If $E_r < 0$ and the "work" integral on the right-hand side diverges as $r_f \to 0$, then the particles will bounce. This occurs for $s = 1, 2$ (cylindrical or spherical symmetry), since $E_r \sim r^{-s}$ neglecting the "current" contribution in Eq. (5.21). Physically, it requires an infinite amount of energy to compress a charged sphere or cylinder to zero size. For $s = 0$ (planar symmetry), the "work" integral is finite, corresponding to the fact that a finite amount of energy is sufficient to compress a plane of charge to zero size. Indeed numerical simulations (discussed below) confirm that caustic singularities do form for planar symmetry, but not for cylindrical or spherical symmetry. However, we expect that departures from perfect planar symmetry will be important near the would-be caustic. Specifically, small fluctuations of the plane symmetric collapse will grow, and we expect the layer to fragment before a planar caustics occurs. (See App. D for a perturbative analysis supporting this picture.) After the fragmentation, lower-dimensional caustics do not occur by the argument above.

The fact that the "current density" j_r generates a negative E_r depends crucially on the non-vanishing second derivative of $\mathcal{A}_r$ at $r = 0$. In fact, the nonlinear equations have exact solutions

$$A_r = \frac{\mathcal{C}_s r}{t_c - t}, \quad E_r = \frac{\mathcal{C}_s(1-\mathcal{C}_s)r}{(t_c - t)^2}, \tag{5.27}$$

where $\mathcal{C}_s = 1$ or $2/(s+1)$ and t_c is an arbitrary constant. For these solutions $j_r = 0$, and nothing prevents the solutions from collapsing at $t = t_c$. These solutions are

of course unrealistic because of the boundary conditions at $r \to \infty$, but we might worry that they are approximately valid near the origin. However, we expect that the scaling solutions Eq. (5.27) are dynamically avoided. This is because the second derivative of A_r must be negative away from $r = 0$, and this propagates to the origin, making $j_r > 0$ and preventing collapse.

5.4 Numerical Results

We now turn to numerical investigations of the nonlinear equations. We begin with the plane symmetric case ($s = 0$) with no external gravitational potential ($\Phi = 0$). For simplicity we assume the symmetry under reflection $r \to -r$. In this case A_r and E_r are odd functions of r. As the initial condition at $t = 0$ we set

$$A_r|_{t=0} = 2\Omega r e^{-r^2}, \quad E_r|_{t=0} = 0, \tag{5.28}$$

where Ω is a positive constant. By performing numerical simulations, we find that for some values of αg^2 and Ω the would-be caustic bounces, but much later it recollapses and forms a caustic that exits the realm of validity of the effective field theory before it bounces. This is shown in Fig. 1. For other values of αg^2 and Ω, even the first caustics at $t \sim 1/\Omega$ does not bounce until the system exits the regime of validity of the effective field theory. This confirms the conclusion of the analytical argument above. As discussed there, we do not expect planar caustics to form in the realistic case.

We now turn to systems with cylindrical and spherical symmetry. Figs. 2 and 3 show results of numerical simulation in cylindrical and spherical cases with $\Phi = 0$, respectively. It is evident that the excitation near the origin diffuses and the system asymptotically approaches the trivial solution $A_r, E_r = 0$. These results confirm our expectation that caustics with cylindrical or spherical symmetry do not occur.

So far, we have been working in the absence of gravity. Let us now consider effects of an external gravitational potential. We still neglect gravity generated by excitations of the gauged ghost condensate. For simplicity we assume spherical symmetry ($s = 2$). A typical numerical result is shown in Figs. 4 and 5. We see that in the final stationary configuration, the repulsive "electric force" E_r cancels the attractive gravitational force $\partial_r \Phi$ (*i.e.* $A_r \to 0$, $E_r \to -\partial_r \Phi$).

6 Black hole accretion

In this section we analyze perturbations of Schwarzschild geometry and estimate the rate of mass increase of the black hole due to accretion of the gauged ghost condensate,

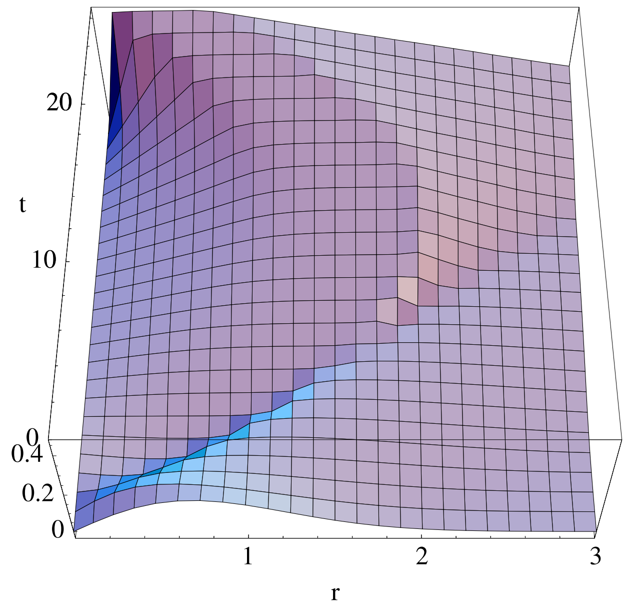


Fig. 1. A result of numerical simulation for planar symmetry ($s = 0$). The variable A_r is plotted as a function of r and t. After an initial bounce, a caustic singularity forms at late times. Note that in a realistic situation, we do not expect a system to exhibit planar symmetry.

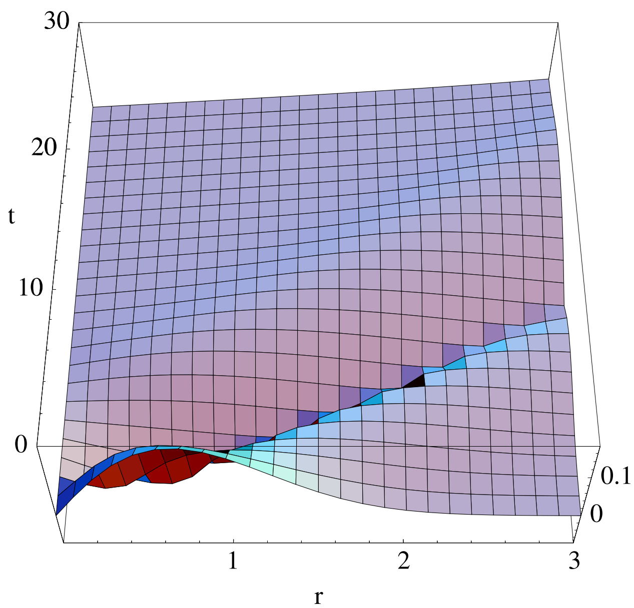


Fig. 2. A result of numerical simulation in cylindrial symmetry ($s = 1$). The variable A_r is plotted as a function of r and t. After an initial bounce, no caustics form at late times.

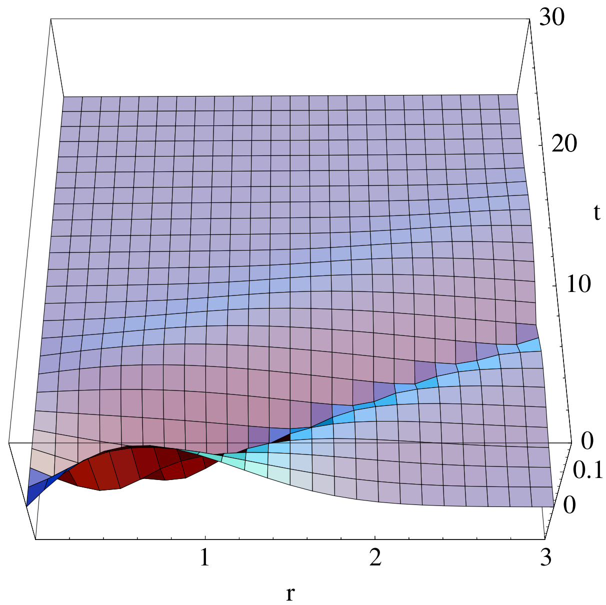


Fig. 3. A result of numerical simulation in spherical symmetry ($s = 2$). The variable A_r is plotted as a function of r and t. Like the case of cylindrical symmetry, there are no late time caustics.

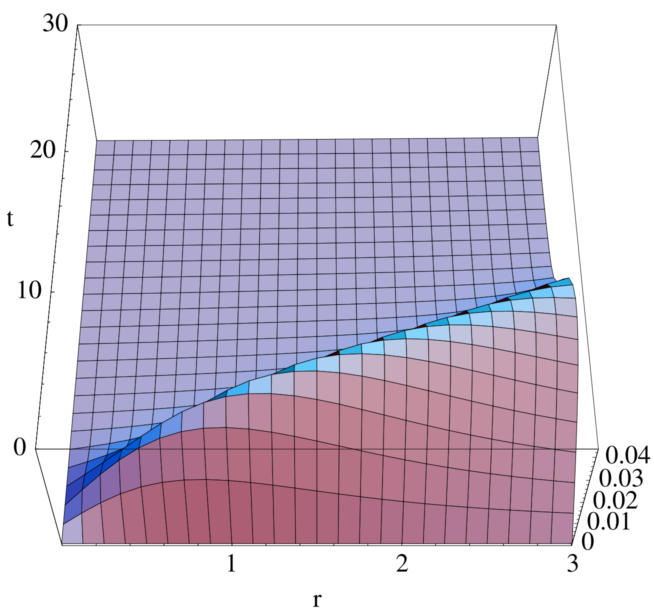


Fig. 4. A result of numerical simulation in spherical symmetry ($s = 2$) with an external gravitational potential. The variable A_r is plotted as a function of r and t. Again, we observe the absence of late time caustics.

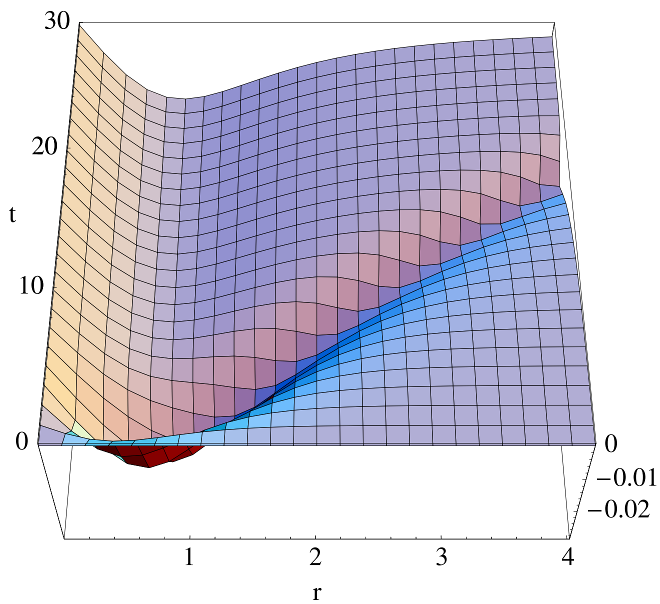


Fig. 5. A result of numerical simulation in spherical symmetry ($s = 2$) with an external gravitational potential. The variable E_r is plotted as a function of r and t. At late times, the "electric field" cancels the gravitational force, halting the accretion of the gauged ghost condensate and protecting the system from caustics.

extending the analysis of Ref. [35] for the ungauged ghost condensate.

Before starting the discussion of black holes, let us first consider a spherically symmetric, static star surrounded by a gauged ghost condensate. In the previous section we have seen that the gauged ghost condensate with spherical symmetry quickly approaches a regular, stationary configuration with or without an external gravitational potential. In the fluid picture, each fiducial fluid particle follows "Newton's law" and approaches the center within the dynamical time set by the external gravitational potential. Near the center, the "electric field" E_r builds up and the infalling flow of fluid particles bounces. Part of outgoing flow produced by the bounce may come back to the center at late time but bounces again. After several (if not one) bounces, the system settles down to a regular, stationary configuration. Inclusion of gravitational backreaction just induces the "renormalization" of Newton's constant (see Eq. (4.12) for the renormalization in the linear theory) and does not change this qualitative behavior towards the stationary configuration since the stress-energy tensor of the final stationary configuration is time-independent and does not have any time-space components. Note that for the ghost fluid around a star to settle down to the stationary configuration, the existence of a regular center is crucial: bounces happen because of the boundary condition at the center.

The ghost condensate surrounding a black hole behaves very differently from that around a star. This is because the boundary condition at the horizon of a black hole is completely different from that at the center of a star. Fiducial fluid particles just go through the horizon and continue to infall. The flow of ghost fluid should still be regular from the viewpoint of infalling comoving observers: shell-crossing type singularities are avoided by bounces due to temporal build-up of the local "electric field". Thus, if we neglect the gravitational backreaction, the ghost condensate surrounding a black hole should approach a congruence of free-falling geodesics in the fluid picture and this should happen roughly within the dynamical (or Kepler) time. The stress energy tensor vanishes if $\alpha = 0$ and is suppressed by the factor $\alpha M^2/M_{Pl}^2$. Hence, the gravitational backreaction does not change this qualitative behavior happening within the dynamical time. Thus, the congruence of free-falling geodesics is a very good approximation to the full solution including gravitational backreaction. However, if we are interested in physics of much longer time scales, the gravitational backreaction may build up to appreciable amount. Therefore, we would like to estimate the mass increase of a black hole due to accretion of the ghost condensate. In the following, we first find the approximate solution representing the congruence of free-falling geodesics and then estimate the gravitational backreaction by perturbative expansion with respect to $\alpha M^2/M_{Pl}^2$.

In order to investigate the behavior of the gauged ghost condensate in a black hole background, we need to choose the time variable carefully. For example, if we use the background Killing time as a time variable then there would appear infinite blue-shift on a black hole horizon. With the infinite blue-shift, the local physical energy scale near the horizon would easily go beyond the scale M. In particular, we would not be allowed to integrate out the field A_0, which has an order-1 overlap with the massive mode in Eq. (3.21). For this reason we should use a different time coordinate for which there is no large blue-shift. In the following analysis we shall use an infalling Gaussian normal coordinate system, in which there is no blue-shift since the time-time component of the metric is -1 everywhere by definition. In this coordinate we can safely use the effective theory with A_0 integrated out. As explained in the previous paragraph, the gauged ghost condensate surrounding a black hole quickly settles down to a congruence of infalling geodesics in the fluid picture. In other words, the solution in the zeroth order in $\alpha M^2/M_{Pl}^2$ is given by $\phi = M^2\tau$, where τ is the time coordinate in the infalling Gaussian normal coordinate system.

The unperturbed Schwarzschild metric in the infalling Gaussian normal coordinate called the Lemaitre reference frame [36] is

$$ds^2 = -d\tau^2 + \frac{(2m_0)^3}{r(\tau,x)}dx^2 + r^2(\tau,x)d\Omega^2, \tag{6.1}$$

where

$$r(\tau,x) = 2m_0\left[\frac{3}{2}\left(x - \frac{\tau}{2m_0}\right)\right]^{2/3}. \tag{6.2}$$

There is nothing bad on the future (black hole) horizon and the coordinate system covers everywhere in the region $v > -\infty$, where v is the ingoing Eddington-Finkelstein null coordinate [37]. The metric becomes ill only on the curvature (physical) singularity at $\tau = 2m_0 x$. We consider spherically-symmetric, time-dependent perturbations of the Schwarzschild geometry. We shall still use an infalling Gaussian normal coordinate system:

$$ds^2 = -d\tau^2 + \frac{(2m_0)^3 e^{2\delta_1(\tau,x)}}{r(\tau,x)}dx^2 + e^{2\delta_2(\tau,x)}r^2(\tau,x)d\Omega^2, \tag{6.3}$$

where $r(\tau,x)$ is given by (6.2). We consider δ_1 and δ_2 as perturbations. Up to the linearized level, the Misner-Sharp energy [38] is expanded as

$$M_{MS} = \frac{e^{\delta_2} r M_{Pl}^2}{2}\left[1 - \partial^\mu(e^{\delta_2}r)\partial_\mu(e^{\delta_2}r)\right] = M_{Pl}^2(m_0 + m_1), \tag{6.4}$$

where

$$\frac{m_1}{r} = \delta_1 - 2m_0\left(\frac{r}{2m_0}\right)^{1/2}\partial_\tau\delta_2 - \left(\frac{r}{2m_0}\right)^{3/2}\partial_x\delta_2 - \left(1 - \frac{3m_0}{r}\right)\delta_2. \tag{6.5}$$

Note that the Misner-Sharp energy agrees with the ADM energy at large distances and, thus, measures the strength of gravity. In the following we shall eliminate δ_1 from all equations by using this expression and consider m_1 and δ_2 as dynamical variables. As for the gauged ghost condensate, we introduce linear perturbation $A(\tau, x)$ as $A_i dx^i = A(\tau, x)dx$. Einstein's equation gives a set of coupled partial differential equations for the three variables m_1, δ_2 and A. Fortunately, it is possible to decouple δ_2 from equations for m_1 and A by taking the following linear combinations of the linearized Einstein tensor:

$$\begin{aligned} (2m_0)^2 \left(\frac{r}{2m_0}\right)^{3/2} \left[G_{\perp\perp} + \frac{G_{\perp x}}{2m_0}\right] &= \frac{\partial_x m_1}{m_0}, \\ (2m_0)^3 \left(\frac{r}{2m_0}\right)^{1/2} \left[G^{xx} + \frac{r^2}{(2m_0)^5} G_{\perp x}\right] &= \frac{\partial_\tau m_1}{m_0}. \end{aligned} \tag{6.6}$$

By applying the formula in Appendix B to

$$\begin{aligned} N = 1, \quad \beta &= 0, \\ q_{ij} dx^i dx^j &= \frac{(2m_0)^3 e^{2\delta_1(\tau,x)}}{r(\tau,x)} dx^2 + e^{2\delta_2(\tau,x)} r^2(\tau,x) d\Omega^2, \\ A_i dx^i &= A(\tau,x)dx, \end{aligned} \tag{6.7}$$

we can obtain the stress energy tensor of the gauged ghost condensate.

In this setup, we expect that accretion of condensate to a black hole is due to the α term. Hence, we perform a perturbation analysis with respect to $\alpha M^2/M_{Pl}^2$, considering m_1, δ_2 and A as quantities of the first order in the perturbation. At zeroth order in $\alpha M^2/M_{Pl}^2$, all components of the stress energy tensor vanish and the Einstein equation is automatically satisfied. This means that there is no mass increase when $\alpha M^2/M_{Pl}^2 = 0$.

At first order in $\alpha M^2/M_{Pl}^2$, the non-vanishing components of the stress energy tensor are

$$\begin{aligned} T_{\perp\perp} &= -\frac{M^2}{(2m_0)^2} \left[\frac{1}{g^2} \frac{r}{2m_0} \partial_\tau \partial_x A + \frac{5}{2g^2} \left(\frac{2m_0}{r}\right)^{1/2} \partial_\tau A + \frac{9\alpha}{8} \left(\frac{2m_0}{r}\right)^3\right], \\ T_{\perp x} &= \frac{9\alpha M^2}{8m_0} \left(\frac{2m_0}{r}\right)^3, \\ T^{xx} &= -\frac{9\alpha M^2}{8(2m_0)^4} \left(\frac{2m_0}{r}\right)^2, \\ T^{\theta\theta} &= -\frac{9\alpha M^2}{8(2m_0)^4} \left(\frac{2m_0}{r}\right)^5. \end{aligned} \tag{6.8}$$

Hence, we obtain

$$T_{\perp\perp} + \frac{T_{\perp x}}{2m_0} = -\frac{M^2}{(2m_0)^2}\left[\frac{1}{g^2}\frac{r}{2m_0}\partial_\tau\partial_x A + \frac{5}{2g^2}\left(\frac{2m_0}{r}\right)^{1/2}\partial_\tau A - \frac{9\alpha}{8}\left(\frac{2m_0}{r}\right)^3\right],$$

$$T^{xx} + \frac{r^2}{(2m_0)^5}T_{\perp x} = -\frac{9\alpha M^2}{8(2m_0)^4}\left(\frac{2m_0}{r}\right)^2\left(1-\frac{r}{m_0}\right). \tag{6.9}$$

Thus, Einstein equation implies that

$$\frac{1}{g^2}\frac{r}{2m_0}\partial_\tau\partial_x A + \frac{5}{2g^2}\left(\frac{2m_0}{r}\right)^{1/2}\partial_\tau A = \frac{9\alpha}{8}\left(\frac{2m_0}{r}\right)^3 - \frac{M_{Pl}^2}{M^2}\left(\frac{2m_0}{r}\right)^{3/2}\frac{\partial_x m_1}{m_0}, \tag{6.10}$$

and

$$\partial_\tau m_1 = -\frac{9\alpha M^2}{16M_{Pl}^2}\left(\frac{2m_0}{r}\right)^{3/2}\left(1-\frac{r}{m_0}\right). \tag{6.11}$$

The first equation should be considered as an equation for A for a given m_1. The second equation does not include A and, thus, can be easily solved w.r.t. m_1. The general solution for m_1 is

$$\frac{m_1}{m_0} = \frac{9\alpha M^2}{4M_{Pl}^2}\left[-\frac{r}{2m_0} + \frac{1}{2}\ln\left(\frac{r}{2m_0}\right) + \bar{C}(x)\right], \tag{6.12}$$

where $\bar{C}(x)$ is an arbitrary function. Note that τ parameterizes each geodesic as the proper time measured by a comoving observer and x parameterizes the congruence of geodesics. Thus, the function $\bar{C}(x)$ represents difference between evolution along different infalling geodesics and thus the initial condition on an initial spacelike hypersurface.

This result is equivalent to the corresponding expression (22) in [35] for the ungauged ghost condensate. To see the equivalence, notice that x here corresponds to x_+ in [35]. Thus, the accretion rate up to first order in $\alpha M^2/M_{Pl}^2$ is the same as that in the ungauged case. In particular, as shown in [35], the leading late-time behavior is

$$\frac{m_1}{m_0} \sim \frac{9\alpha M^2}{4M_{Pl}^2}\left(\frac{3v}{4m_0}\right)^{2/3}, \tag{6.13}$$

where v is the advanced null time coordinate normalized at infinity (the ingoing Eddington-Finkelstein null coordinate). To obtain this formula, the asymptotic behavior of the function $\bar{C}(x)$ for large x has been determined by the assumption that the initial value of m_1 on an initial spacelike hypersurface does not diverge at large r, and the limit $v \gg r$ has been taken. This formula is valid in the regime where m_1 is sufficiently small compared to m_0. In order to see the behavior beyond this regime,

let us "renormalize" the parameter m_0. For this purpose we note that

$$\frac{1}{M_{Pl}^{4/3}}\frac{d(M_{BH}^{2/3})}{d(v^{2/3})} \sim 2\left(\frac{3}{4}\right)^{5/3}\frac{\alpha M^2}{M_{Pl}^2} \tag{6.14}$$

for the black hole mass $M_{BH} = (m_0 + m_1)M_{Pl}^2$. This gives the solution

$$M_{BH}^{2/3} \sim M_0^{2/3} + 2\left(\frac{3}{4}\right)^{5/3}\frac{\alpha M^2}{M_{Pl}^2}(M_{Pl}^2 v)^{2/3} \tag{6.15}$$

for the mass M_{BH} surrounded by the sphere at the radius r at the time v, where M_0 is the initial value of M_{BH} at $v = 0$. This formula is valid if the condition $v \gg r$ is satisfied. We shall check this condition when we apply this formula to a stellar-mass black hole below.

Convincing evidences for stellar-mass black holes are provided by X-ray binaries, such as Cygnus X1. Some candidates for the stellar-mass black holes are listed in Table 1 of [39]. On the other hand, ages of those stellar-mass BHs are less certain. For example, XTE J1118+480 is thought to be a black hole with mass $M_{BH} \sim 7M_\odot$ and binary separation $r \sim 3R_\odot$, but its age is estimated to be either ~ 240 Myr or ~ 7 Gyr [40], depending on whether it was kicked by a supernova explosion or was ejected from a globular cluster. (GRO J1655-40 has $M_{BH} \sim 5M_\odot$ and its age is estimated to be ~ 0.7 Myr [41]. This would give a weaker bound.)

The observation of XTE J1118+480 implies that the mass measured at $r \sim 3R_\odot$ is $M_{BH} \sim 7M_\odot$ at $t \sim 240$ Myr or ~ 7 Gyr, while the theory of stellar evolution says that M_0 must be larger than $\sim 3M_\odot$. Note that the estimate of t should not change significantly even if M_{BH} actually evolved from $\sim 3M_\odot$ to $\sim 7M_\odot$ during its journey. We can apply the formula (6.15) to this system since $M_{Pl}^2 r/M_{BH} \sim 10^5$ and $M_{Pl}^2 t/M_{BH} \sim 10^{20}$ and thus the condition $v \sim t \gg r$ is satisfied. Therefore, we obtain

$$(7M_\odot)^{2/3} - (3M_\odot)^{2/3} \gtrsim \frac{\alpha M^2}{M_{Pl}^2}(M_{Pl}^2 \times 240 \text{ Myr})^{2/3}. \tag{6.16}$$

This gives the bound [8] on M as

$$\sqrt{\alpha}M \lesssim 10^{12} \text{ GeV}. \tag{6.17}$$

[8] Because of the motion of the binary system, the relative rotation of the binary components and the non-zero spin of the black hole, the spherical symmetry assumed in the derivation of the formula (6.15) is violated. It is expected that these factors will increase the accretion rate. Thus, while (6.17) is valid as an upper bound on $\sqrt{\alpha}M$, it is worthwhile seeking more stringent bound by taking them into account.

7 Čerenkov Radiation from Gravitating Sources

For a gravitational source moving faster than the sound velocity of the ether ($v > c_s$) ether Čerenkov radiation will be emitted. As we will see, the energy loss per unit time is at most of order $M^2 v^3$, where M is the scale of the gauge ghost condensate, independently of the size of the source. In the ungauged ghost condensate, the same result holds [9], but in the gauged case M can be much larger, which give rise to interesting phenomena and constraints from this effect.

Consider a non-relativistic source

$$\mathcal{L}_{\mathcal{S}} = -\frac{1}{\sqrt{2}M_{\rm Pl}}\rho\,\Phi^c. \tag{7.1}$$

As shown in App. E, the energy loss rate can be calculated as

$$\dot{E}_{\rm src} = \int d^3x \frac{1}{\sqrt{2}M_{\rm Pl}}\rho(x,t)\dot{\Phi}^c(x,t). \tag{7.2}$$

At the linearized level,

$$\tilde{\Phi}^c(\omega,k) = \left[-\frac{1}{k^2} + \frac{\alpha M^2}{2M_{\rm Pl}^2}\frac{1}{\omega^2 - c_s^2 k^2}\right]\frac{\tilde{\rho}(\omega,k)}{\sqrt{2}M_{\rm Pl}}, \tag{7.3}$$

where we have ignored the k^4 piece in the denominator of the second term in the bracket because it is never important for the large astrophysical bodies we consider here.

For a star of mass $M_\odot$ moving on a straight line with velocity $\vec{v}$:

$$\tilde{\rho}(\omega,k) = 2\pi M_\odot \delta(\omega - \vec{k}\cdot\vec{v}) f(k), \tag{7.4}$$

where $f(k)$ is the dimensionless form factor given by the Fourier transform of the mass density distribution. For example, a uniform-density sphere of radius $R_\odot$ gives

$$f(k) = \frac{3(\sin kR_\odot - kR_\odot \cos kR_\odot)}{(kR_\odot)^3}. \tag{7.5}$$

Substituting Eq. (7.4) into Eqs. (7.2) and (7.3), we have

$$\dot{E}_{\rm src} = \frac{M_\odot^2}{2M_{\rm Pl}^2}\int \frac{d^3k}{(2\pi)^3}(-i\vec{k}\cdot\vec{v})\left[-\frac{1}{k^2} + \frac{\alpha M^2}{2M_{\rm Pl}^2}\frac{1}{(\vec{k}\cdot\vec{v} + i\varepsilon)^2 - c_s^2 k^2}\right]|f(k)|^2, \tag{7.6}$$

where we have used the $i\epsilon$ prescription for the retarded Green's function. Note that the integral is only nonzero due to the poles of the second term, *i.e.* when the propagator is on shell. This occurs only for $v > c_s$, and we obtain

$$\dot{E}_{\rm src} = -\frac{\alpha M_\odot^2 M^2}{16\pi M_{\rm Pl}^4 v}\int_0^\infty k\,dk|f(k)^2|, \tag{7.7}$$

for $v \gg c_s$. For a source of size $R_\odot$, we expect

$$\int_0^\infty k\,dk|f(k)^2| \sim \frac{1}{R_\odot^2}. \tag{7.8}$$

However, as we discussed in the previous section, nonlinear effects can be important near a gravitational source, and may cut off the integral Eq. (7.7) at radius larger than $R_\odot$. Using the estimates of the previous section, we see that nonlinear effects become important within a distance

$$r \lesssim R_{\rm nonlinear} \sim \frac{M_\odot}{M_{\rm Pl}^2 c_s^2}. \tag{7.9}$$

In the ungauged ghost condensate, the parametric formula for the rate of energy loss in the linear regime continues to be valid in the nonlinear regime [10], and we find the same result here for essentially the same reason. Consider a fluid particle moving past a source with speed v and impact parameter r. In the impulse approximation, a fluid particle gets a kick perpendicular to its initial velocity due to gravity

$$\Delta v_\perp \sim F\Delta t \sim \frac{M_0}{M_{\rm Pl}^2 v r}. \tag{7.10}$$

Here we have neglected the "electric" force on the particle. This can be estimated using

$$\frac{1}{M}E_\perp \sim \frac{\alpha g^2 \Delta v_\perp}{r^2}\Delta t \sim \frac{\alpha g^2 M_0}{M_{\rm Pl}^2 v^2 r^2} \sim \frac{c_s^2}{v^2}\vec{\nabla}\Phi. \tag{7.11}$$

We see that neglecting the "electric force" is justified for $v > c_s$. The impulse approximation is valid as long as the perpendicular distance travelled by the fluid particle is less than r, which gives

$$r \lesssim R_{\rm drag} \sim \frac{M_0}{M_{\rm Pl}^2 v^2}. \tag{7.12}$$

Note that $R_{\rm drag} < R_{\rm nonlinear}$ for $v > c_s$. For $r \lesssim R_{\rm drag}$, we expect the fluid to be "dragged" with the source, and no Čerenkov radiation will be emitted from the dragged region.

The energy loss induces an anomalous acceleration in the direction of the ether wind,

$$a_{\rm anom} = \frac{\dot{E}_{\rm src}}{M_\odot v} \simeq \frac{\alpha\kappa M_\odot M^2}{16\pi M_{\rm Pl}^4 v^2 R_\odot^2}. \tag{7.13}$$

Because it is proportional to $M_\odot/R_\odot^2$, the Sun gets the largest anomalous acceleration in the solar system. These accelerations will modify the orbits of the planets.

In particular, they will cause misalignments of the orbital planes if the anomalous accelerations are out of the ecliptic plane. For comparison, the typical relative accelerations between the planets and the Sun are $10^{-34} - 10^{-38}$ GeV, the Viking ranging data constrains any anomalous radial acceleration acting on Earth or Mars to be smaller than $\sim 10^{-44}$ GeV while the Pioneer anomaly corresponds to $a_{\rm Pioneer} \sim 10^{-42}$ GeV [42]. If we require the anomalous accelerations to be smaller than $\sim 10^{-44}$ GeV, we obtain an upper bound on M:

$$M \lesssim 10^{10}(\alpha\kappa)^{-1/2}\ \text{GeV}. \tag{7.14}$$

Another potential constraint outside the solar system comes from the period change of binary pulsar systems. Note that even if the velocities of the pulsars are smaller than c_s, the Goldstone boson can still be emitted through the quadrupole radiation. To simplify the problem, we assume that two pulsars of equal mass M_0 move in a circular orbit separate by a distance $2r_0$ with an angular frequency ω_0. The angular frequency and the orbital velocity v_0 are related by $v_0 = \omega_0 r_0$. We also ignore the velocity of the center of the system.

The energy loss due to the multipole radiation in the case where $v_0 < c_s$ or Čerenkov radiation in the case where $v_0 > c_s$ can be calculated in the same way described in App. E. Analytic formulae can be obtained by taking different limits of the Bessel functions for $v_0 \ll c_s$ or $v_0 \gg c_s$.

For $v_0 = \omega_0 r_0 \ll c_s$, the energy loss is dominated by the quadrupole radiation. As derived in App. E, it is given by Eq. (E.13).

$$\dot{E}_{\rm src} = -\frac{4\alpha M_0^2 M^2 v_0^6}{15\pi M_{\rm Pl}^4 r_0^2 c_s^7} = -\frac{2^{12}\pi\alpha M^2 v_0^{10}}{15\, c_s^7}. \tag{7.15}$$

On the other hand, the energy loss due to usual gravitational waves is given by [31]

$$\dot{E}_{\rm tensor} = -\frac{1}{8\pi M_{\rm Pl}^2}\frac{1}{5}\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\rangle = -\frac{2}{5}\,\frac{G_N^4 M_0^5}{r_0^5} = -\frac{2^{14}\pi M_{\rm Pl}^2 v_0^{10}}{5}, \tag{7.16}$$

where Q_{ij}'s are the quadrupole moments of the gravitational source. Taking the ratio, we have

$$\frac{\dot{E}_{\rm src}}{\dot{E}_{\rm tensor}} = \frac{\alpha M^2}{12 M_{\rm Pl}^2 c_s^7} = \frac{\alpha_2^{\rm PPN}}{6\, c_s^3} \simeq 10^2\left(\frac{\alpha_2^{\rm PPN}}{4\times 10^{-7}}\right)\left(\frac{10^{-3}}{v_0}\right)^3\left(\frac{v_0}{c_s}\right)^3. \tag{7.17}$$

The observed period change of the binary pulsars PSR 1913+16 is consistent with the energy loss entirely due to the gravitational wave radiation [31], so the above ratio must be smaller than 1. Given their orbital velocity $v_0 \sim 10^{-3}$, we can see that for

$v_0 \ll c_s$ (say, $c_s > 10\, v_0$), this does not give a stronger bound than the $\alpha_2^{\rm PPN}$ bound from the solar system. For v_0 getting close to c_s, this can compete with the solar system bound, but our approximation starts to break down.

Next, let us consider the opposite limit, $c_s \ll v_0$. The energy loss is given approximately by (see Eq. (E.15) of App. E)

$$\dot{E}_{\rm src} \simeq -\frac{\alpha M^2 M_0^2}{4\pi M_{\rm Pl}^4 c_s r_0^2} \sum_{n=1}^{\infty} \frac{2nv_0}{c_s} \left| f\left(\frac{2nv_0}{c_s r_0}\right)^2 \right|. \tag{7.18}$$

The problem is that the arguments in the form factor take values much bigger than $1/r_0$ in this case. As we discussed before, nonlinear effects are already important at that length scale and the region of the ether being dragged by the pulsars is approximately of size r_0. The effective form factor must be very suppressed and is difficult to calculate from first principles. Physically, the binary system emits ether waves with angular frequencies that are integer multiples of $2\omega_0$. Because the sound velocity of the ether is much smaller than the orbital velocity, the wavelengths of the ether waves are much shorter than the size of the system. Therefore, the amplitudes of the ether waves must be highly suppressed. If we assume a power law suppression that $f(2nv_0/c_s r_0) \sim (2nv_0/c_s)^{-p}$ with $p > 1$, then the sum in Eq. (7.18) is $\sim (c_s/v_0)^{2p-1}$, and the energy loss would be

$$\dot{E}_{\rm src} \sim -\frac{\alpha M^2 M_0^2}{4\pi M_{\rm Pl}^4 r_0^2 c_s} \left(\frac{c_s}{v_0}\right)^{2p-1} \sim 10^3 \alpha \left(\frac{M}{10^{10}\ {\rm GeV}}\right) \left(\frac{10^{-3}}{v_0}\right)^7 \left(\frac{c_s}{v_0}\right)^{2p-2} \frac{dE_{\rm tensor}}{dt}. \tag{7.19}$$

Obviously for $c_s \ll v_0$ this does not give a significant constraint. For c_s getting close to v_0 it may become a competitive constraint depending on the power of the suppression factor, but again our approximation starts break down.

8 Couplings to the Standard Model

Given that gauged ghost condensation yields such a modest modification of gravity, the strongest bounds on this model of Lorentz breaking could come from direct couplings between $\mathcal{A}_\mu$ and the standard model. As we will see, most couplings could be forbidden by discrete symmetries, but we will consider what would happen if the gauged ghost condensate were not confined to a hidden sector.

Even in the absence of gravitational couplings, the Lagrangian in Eq. (3.23) is fascinating because it is the effective field theory description of an Abelian gauge theory in a Lorentz-violating Higgs phase. To our knowledge, the dynamics of this phase has not been studied in the literature. Like the Lorentz-invariant Higgs phase,

the gauge boson eats a Goldstone and therefore has three polarizations, but what is especially bizarre about this system is that all three polarizations are massless. There are long-range "magnetic" interactions in this theory but "electric" interactions are exponentially suppressed. In other words, in order to produce a healthy Newton's law for gravity, we had to sacrifice Coulomb's law for this ghost-electromagnetic $U(1)$.

Note that this $U(1)_{\text{ghost}}$ theory is different from other Lorentz-violating $U(1)$ gauge theories considered in the literature. In theories based on Ref. [1], the Lagrangian contains gauge invariant but Lorentz-violating terms like $k_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$, where $k_{\alpha\beta\gamma\delta}$ is some fixed Lorentz-violating tensor. Because gauge symmetry is maintained, the equations of motion for electric and magnetic fields are modified without introducing new propagating degrees of freedom. A model that does violate gauge symmetry is given in Ref. [43] which considers a "magnetic" Higgs phase where Coulomb's law is virtually unchanged. Starting with the proposal of Ref. [44], there has also been speculation that the electromagnetic field could arise as Goldstone bosons from spontaneous Lorentz breaking. In contrast to these other theories, gauged ghost condensation is a model where a $U(1)$ gauge field enters an "electric" Higgs phase triggered by a Lorentz-violating vev for a charged scalar field.

8.1 Catalog of Allowed Couplings

At minimum, even if we do not include direct interactions between standard model fields and $\mathcal{A}_\mu$, we expect graviton loops to generate interactions of the form

$$\Delta\mathcal{L} \sim c\frac{M^2}{M_{\text{Pl}}^4}T^{\mu\nu}\mathcal{A}_\mu\mathcal{A}_\nu, \tag{8.1}$$

where $T_{\mu\nu}$ is a symmetric dimension four standard model operator, and c is expected to be $\mathcal{O}(1)$. Note that there is no constraint from gauge invariant on this coupling because $\mathcal{A}_\mu = A_\mu + M\partial_\mu\phi$ is a $U(1)_{\text{ghost}}$ gauge invariant combination. The $1/M_{\text{Pl}}^4$ suppression arises because $\mathcal{A}_\mu$ contains $\partial_\mu\phi$ so this interaction is actually dimension eight, and the factors of M are inserted because $\mathcal{A}_\mu$ is defined with mass dimension +1. Setting $\mathcal{A}_\mu$ to its vacuum expectation value $\langle\mathcal{A}_0\rangle = M$:

$$\Delta\mathcal{L} \sim c\frac{M^4}{M_{\text{Pl}}^4}T^{00}. \tag{8.2}$$

If we take $T^{\mu\nu}$ to be the stress-energy tensor for a standard model fermion, the inclusion of T^{00} allows the maximal attainable velocity of the fermion to differ from the speed of light:

$$\delta v_\psi \sim \frac{M^4}{M_{\text{Pl}}^4}. \tag{8.3}$$

Constraints from high precision spectroscopy and the absence of vacuum Čerenkov radiation bound $\delta v_\psi \lesssim 10^{-21} - 10^{-23}$ [45]. This limits the scale of spontaneous Lorentz violation to be $M \lesssim 10^{13}$ GeV. Remarkably, for $g \sim 1/10$ this bound on M is comparable to the bound from the gravitational sector in Sec. 4.

If there is a $\mathcal{A}_\mu \to -\mathcal{A}_\mu$ symmetry, then Eq. (8.1) is the leading coupling between the gauged ghost condensate and the standard model. Relaxing this symmetry allows couplings of the form

$$\Delta\mathcal{L} = \tilde{q} J^\mu \mathcal{A}_\mu, \tag{8.4}$$

where J^μ is a standard model current and $\tilde{q}$ is the coupling constant. In fact, this is just equivalent to charging standard model fields under the $U(1)_{\rm ghost}$ gauge symmetry. To see this, consider the interaction

$$\tilde{q}_\psi\, \mathcal{A}_\mu \bar{\psi}\gamma^\mu\psi = \tilde{q}_\psi\, (A_\mu + M\partial_\mu\phi)\bar{\psi}\gamma^\mu\psi, \tag{8.5}$$

where ψ is a standard model fermion. If the current $\bar{\psi}\gamma^\mu\psi$ is conserved, its interaction with $\partial_\mu\phi$ can be removed by a field redefinition

$$\psi \to \psi' = e^{-i\tilde{q}_\psi M\phi}\psi \tag{8.6}$$

into the kinetic term. In terms of the redefined field ψ', the interaction Eq. (8.5) becomes

$$\tilde{q}_\psi A_\mu \bar{\psi}'\gamma^\mu\psi' \tag{8.7}$$

which is just a $U(1)_{\rm ghost}$ gauge interaction of a fermion with charge $\tilde{q}_\psi$. The point is that $\exp(-iM\phi)$ carries a unit charge of the $U(1)_{\rm ghost}$ gauge symmetry, so the charge of a field under $U(1)_{\rm ghost}$ can be altered by multiplying it by powers of $\exp(-iM\phi)$. It is most convenient, however, to work in the ψ' basis where the $\partial_\mu\phi$ coupling is removed.

New long-range forces other than electromagnetism and gravity are strongly constrained. In gauged ghost condensation the $\mathcal{A}_0$ field is massive, so there is no $1/r$ potential between $U(1)_{\rm ghost}$ charges in the preferred rest frame. Nonetheless, $\mathcal{A}_i$ is still massless, giving rise to additional magnetic forces. Effectively, this gives a tree-level contribution to the magnetic moment of a particle charged under the $U(1)_{\rm ghost}$. If we assume that the $U(1)_{\rm ghost}$ charges of the standard model fields are proportional to their $U(1)_{\rm EM}$ charges, the correction to the Landé g factor is

$$\delta\left(\frac{g-2}{2}\right) \sim \frac{\tilde{q}_e^2\alpha_{\rm ghost}}{\alpha_{\rm EM}}, \tag{8.8}$$

where $\alpha_{\rm ghost} = g^2/(4\pi)$. Note that in theories with a normal para-$U(1)$ gauge group that "regauges" electromagnetism [46], this correction is absent, because the para-$U(1)$ also shifts the electrostatic charges of the fermions. The most accurately measured $g-2$ is for the electron with an uncertainty at the 10^{-9} level [47, 48]. However, since it serves as a definition of $\alpha_{\rm EM}$, to constrain $\tilde{q}_e g$ we need the next most precise determination of $\alpha_{\rm EM}$. A measurement of $\alpha_{\rm EM}$ using the atom interferometry with the laser-cooled cesium atoms has reached the 10^{-8} uncertainty level and the result is in agreement with the $g-2$ measurement [49]. This implies a constraint

$$\frac{\tilde{q}_e^2 \alpha_{\rm ghost}}{\alpha_{\rm EM}} \lesssim 10^{-8} \qquad \Longrightarrow \qquad \tilde{q}_e g \lesssim 10^{-4}. \tag{8.9}$$

As we will see, this bound is much weaker than the bound from the long-range force derived in the next subsection: for sources moving with respect to the preferred frame, there is a direction-dependent pseudo-Coulomb potential. If the $U(1)_{\rm ghost}$ charges are not proportional to the electromagnetic charges, then the measurement of $(g-2)$ would depend on which species of fermion was responsible for setting up the background magnetic field, though in practice, almost all laboratory magnetic fields are in some way established by electrons.

One can also consider the couplings of standard model antisymmetric tensors to the field tensor of the $U(1)_{\rm ghost}$. At the lowest order, we can write down a kinetic mixing between the $U(1)_{\rm ghost}$ and $U(1)_{\rm EM}$,

$$\Delta\mathcal{L} = c_o F_{\mu\nu} F^{\mu\nu}_{\rm EM}. \tag{8.10}$$

Such a term can be removed by a redefinition of the $U(1)$ gauge fields, resulting in a shift of the $U(1)_{\rm ghost}$ charges of the standard model fields, so the effect is the same as discussed in the previous paragraph. Alternatively, we could imagine dipole interactions

$$\Delta\mathcal{L} = \frac{1}{\Lambda} \sum_{\Psi} c_\Psi \bar{\Psi} \sigma^{\mu\nu} \Psi F_{\mu\nu}. \tag{8.11}$$

These couplings were studied in a Lorentz invariant setting in Ref. [50], where the bound on Λ for the electron was expressed in terms of the Higgs vev $\langle h \rangle = 174$ GeV:

$$\frac{\Lambda}{c_e} > \frac{(3-60 \text{ TeV})^2}{\langle h \rangle} \sim 10^5 - 10^7 \text{ GeV}, \tag{8.12}$$

where the more stringent bound comes if Eq. (8.11) introduces flavor mixing between electrons and muons. It would be interesting to study these couplings more carefully

in the Lorentz-violating Higgs phase, but we do not expect significant departures from these bounds.

Because we will find strong bounds on $\tilde{q}_\psi$ from direction dependent pseudo-Coulomb potentials, we want to forbid the coupling in Eq. (8.5) while still allowing new couplings other than just Eq. (8.1). One way to forbid a coupling to the electromagnetic current is to choose ϕ to be parity-odd, or equivalently

$$P: \qquad \mathcal{A}_0 \to -\mathcal{A}_0, \qquad \mathcal{A}_i \to \mathcal{A}_i. \tag{8.13}$$

In the standard model, electroweak interactions violate parity, and because we expect the scale M to be much higher than the electroweak scale, at best we can say that the coupling of $\mathcal{A}_\mu$ to parity-even currents should be suppressed relative to parity-odd currents. For a fermion Ψ with Dirac mass m_D, we can construct the vector and axial currents:

$$J_\mu = \bar{\Psi}\gamma_\mu\Psi, \qquad J^5_\mu = \bar{\Psi}\gamma_\mu\gamma^5\Psi. \tag{8.14}$$

By looking at the relevant triangle diagram, we estimate that the coupling $\mathcal{A}^\mu J_\mu$ should be suppressed relative to the coupling $\mathcal{A}^\mu J^5_\mu$ by $m_D^2 G_F/16\pi^2$, where G_F is Fermi's constant. For an electron, this suppression is 10^{-14}, so to a good approximation we can ignore couplings to parity-even currents.

With the above caveats, the leading coupling of $\mathcal{A}_\mu$ to any light Dirac fermion, in particular electrons and nucleons, is

$$\mathcal{L}_{\text{int}} = \frac{M}{F}\mathcal{A}^\mu J^5_\mu, \tag{8.15}$$

where we parametrize the coefficient as a ratio of two scales M and F to compare with the result of ungauged ghost condensation. Setting $\mathcal{A}_\mu$ to its vev:

$$\mathcal{L}_{\text{int}} = \mu\bar{\Psi}\gamma^0\gamma^5\Psi, \qquad \mu = \frac{M^2}{F}. \tag{8.16}$$

This Lorentz- and CPT-violating term is the same as in ghost condensation [8, 9], and gives rise to different dispersion relations for the left- and right-handed fermions [51, 52]. In a frame boosted with respect to the preferred frame, Eq. (8.16) contains the interaction

$$\mu\vec{s}\cdot\vec{v}_{\text{earth}}, \tag{8.17}$$

where we have identified $\bar{\Psi}\vec{\gamma}\gamma^5\Psi$ with non-relativistic spin density $\vec{s}$. Assuming the preferred rest frame is aligned with the CMBR, we take $|\vec{v}_{\text{earth}}| \sim 10^{-3}$, and experimental bounds on velocity-dependent spin couplings to electrons is $\mu \sim 10^{-25}$ GeV [53] and to nucleons $\mu \sim 10^{-24}$ GeV [54, 55].

After integrating out $\mathcal{A}_0$, we are left with the interaction

$$\mathcal{L}_{\rm int} = \frac{M}{F}\vec{s}\cdot\vec{\mathcal{A}}. \tag{8.18}$$

The $g \to 0$ limit of this coupling was discussed in Ref. [9] and it led to two Lorentz-violating dynamical effects: "ether" Cerenkov radiation and a long-range spin-dependent potential. These effects in this model will be discussed in the next subsection.

8.2 Dynamics of the Lorentz-Violating Higgs Phase

The Lagrangian in Eq. (3.16) is the most general Lagrangian with an $\mathcal{A}_\mu \to -\mathcal{A}_\mu$ symmetry that is spontaneously broken by the vacuum $\langle\mathcal{A}_0\rangle = M$. If we only enforce Eq. (8.13), then we can add additional couplings to the gauged ghost Lagrangian such as

$$\Delta\mathcal{L} \sim M(\mathcal{A}_0 - M)\partial_i\mathcal{A}^i. \tag{8.19}$$

When we integrate out $\mathcal{A}_0$, however, this term does not change the basic form of Eq. (3.23), and the effective Lagrangian for $\mathcal{A}_i$ at distances large compared to $1/M$ is

$$\mathcal{L}_{\rm eff} = \frac{1}{2c_v^2g^2}(\partial_t\vec{\mathcal{A}})^2 - \frac{1}{4g^2}F_{ij}^2 - \tfrac{1}{2}\alpha(\vec{\nabla}\cdot\vec{\mathcal{A}})^2, \tag{8.20}$$

where we have absorbed the effect of β_1 and α_2 into the parameters g and c_v, and c_v has the interpretation of the velocity of the transverse vector modes.

Going to $\mathcal{A}_i \equiv A_i$ gauge, it is straightforward to calculate the A_iA_j propagator,

$$\langle A_iA_j\rangle = \frac{1}{(\omega/c_v)^2 - k^2}\left(g^2\delta_{ij} + (g^2 - 1/\alpha)\frac{k_ik_j}{(\omega/c_s)^2 - k^2}\right), \tag{8.21}$$

where $c_s = c_v g\sqrt{\alpha}$ is the velocity of the longitudinal mode of A_i. We will always assume that $c_s \ll c_v$. The first thing to check is what kind of pseudo-Coulomb potential is generated between two particles carrying $U(1)_{\rm ghost}$ charges in a frame moving with respect to the preferred frame. Consider a source of charge $\tilde{Q}$ under $U(1)_{\rm ghost}$ moving with velocity $v < c_s$:

$$\begin{aligned} J^0 &= \tilde{Q}\delta^{(3)}(\vec{x} - \vec{v}t) & \tilde{J}^0 &= 2\pi\tilde{Q}\delta(\omega - \vec{k}\cdot\vec{v}) \\ \vec{J} &= \tilde{Q}\vec{v}\delta^{(3)}(\vec{x} - \vec{v}t) & \tilde{\vec{J}} &= 2\pi\tilde{Q}\vec{v}\delta(\omega - \vec{k}\cdot\vec{v}) \end{aligned} \tag{8.22}$$

where there are corrections at $\mathcal{O}(v^2)$. To this order, we can use the $\omega \to 0$ limit of Eq. (8.21). Taking the Fourier transform of the propagator, the potential between $\tilde{Q}$ and a test charge $\tilde{q}$ a comoving distance r away is

$$V(r,\theta) = -\frac{g^2 v^2 \tilde{Q}\tilde{q}}{8\pi r}\left(1 + \cos^2\theta + \frac{1}{g^2\alpha}\sin^2\theta\right), \tag{8.23}$$

where $\cos\theta = \hat{r}\cdot\hat{v}$. In the limit $c_s \ll c_v$, the third term dominates, yielding an effective anomalous gauge coupling

$$g_{\text{anom}} \sim \frac{v}{\sqrt{\alpha}}, \tag{8.24}$$

and angular dependence with respect to the velocity of the preferred frame. Bounds on new gauge interactions are usually derived by noting the absence of any long-range forces other than electromagnetism and gravity. In particular, precision tests of gravity using materials with different baryon-to-lepton ratios place limits on a possible $U(1)_{B-L}$ gauge coupling of $g_{B-L} < 10^{-23}$ [56]. If searches for $U(1)_{B-L}$ gauge couplings are sensitive to such velocity effects, then, $\tilde{q}g_{\text{anom}} \lesssim 10^{-23}$. Assuming the earth is moving with velocity $|\vec{v}| \sim 10^{-3}$ with respect to the preferred rest frame, then

$$\tilde{q}_\psi \lesssim 10^{-20}\sqrt{\alpha}, \tag{8.25}$$

where ψ represents an ordinary particle p, n, or e. For reasonable values of α and g, this bound is stronger than Eq. (8.9), indicating that couplings between $\mathcal{A}_\mu$ and vector currents must be strongly suppressed. Still, it would be interesting to search for the angular dependent Coulomb potential in Eq. (8.23) to see whether the bound on $\tilde{q}_\psi$ could be improved.

Even if we forbid the coupling to vector currents using Eq. (8.13), axial couplings are still allowed. The coupling in Eq. (8.15) between $\vec{A}$ and spin is familiar from ghost condensation and gives rise to a spin-dependent inverse-square law force [9]. Consider spins $\vec{S}_1$ and $\vec{S}_2$ separated by a distance r that are at rest with respect to the preferred frame. Taking the Fourier transform of Eq. (8.21) with $\omega \to 0$, the $1/r$ potential between them takes the form

$$V(r) = -\frac{M^2}{F^2}\frac{1}{8\pi r}\Big((g^2 + 1/\alpha)\vec{S}_1\cdot\vec{S}_2 + (g^2 - 1/\alpha)(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})\Big). \tag{8.26}$$

This reduces to the pure Goldstone result in the $g \to 0$ limit, modulo a redefinition $M \to \sqrt{\alpha}M$. This potential has an interesting feature which distinguishes it from both electromagnetism and ghost condensation. Consider a toroidal solenoid filled

with a ferromagnetic material.[9] When current runs through the solenoid, the ferromagnetic spins will align in the azimuthal direction, but as we will show in App. F, there will be no net magnetic field nor net longitudinal $\vec{A}$ field outside of the solenoid. However, there will be transverse $\vec{A}$ fields which can interact with a test spin, and because there is no magnetic field leakage from this geometry, this allows for a null test for the inverse-square law spin-dependent force.

In order for the potential in Eq. (8.26) to be experimentally testable, it has to be at least roughly of gravitational strength, and if we assume spin sources with one aligned spin per nucleon, M/F has to be of order $m_N/M_{\rm Pl} \sim 10^{-19}$. As shown in Ref. [9], M/F cannot be much larger than 10^{-19} without the theory exiting its range of validity. When M/F is so small, it is easy to satisfy the condition in Eq. (8.25). In the previous section, we argued that the ratio of the vector coupling to the axial coupling can be made as small as $m_e^2 M_{\rm Pl}/16\pi^2 \sim 10^{-14}$ without fine tuning if we assume the quasi-parity invariance in Eq. (8.13). This tells us that

$$\frac{\tilde{q}_\psi}{M/F} < 10^{-14} \qquad \Longrightarrow \qquad \tilde{q}_\psi < 10^{-35} \tag{8.27}$$

in agreement with Eq. (8.25).

This spin-spin potential is even more interesting at finite velocity. For sources moving at velocity $\vec{v}$ will respect to the preferred frame, the potential at fixed co-moving distance is given by the Fourier transform of Eq. (8.21) with the replacement $\omega \to \vec{k} \cdot \vec{v}$. In the case $v < c_s$, we can calculate the spin-spin potential as a power series in v/c_s. For simplicity, we define

$$f(\vec{A}, \vec{B}) = \vec{A} \cdot \vec{B} - (\vec{A} \cdot \hat{r})(\vec{B} \cdot \hat{r}), \tag{8.28}$$

and the potential takes the form

$$\begin{aligned} \frac{F^2}{M^2} V(r) = &-\frac{g^2}{4\pi r} \vec{S}_1 \cdot \vec{S}_2 - \frac{1/\alpha - g^2}{8\pi r} f(\vec{S}_1, \vec{S}_2) \\ &- \frac{1/\alpha - g^2}{32\pi r} \frac{v^2}{c_s^2} \Big(f(\vec{S}_1, \vec{S}_2) f(\hat{v}, \hat{v}) + 2 f(\vec{S}_1, \hat{v}) f(\vec{S}_2, \hat{v}) \Big) + \mathcal{O}(v^4/c_s^4). \end{aligned} \tag{8.29}$$

The unique angular dependence of this spin-spin potential is a smoking gun for gauged ghost condensation. As v gets larger than c_s, the potential should map onto the shadow/shockwave form of Ref. [9], though to see this behavior, one would need to keep track of k^4 terms in the dispersion relation for the $\mathcal{A}_i$ longitudinal mode.

[9] Thanks to Blayne Heckel for suggesting this geometry.

Next, we consider energy loss due to the emission of $\mathcal{A}_i$ particles. In Ref. [9] we did not consider bounds on new light particles from astrophysics [57], because the scale of spontaneous Lorentz violation M was constrained to be much lower than typical astrophysical energies. Now that we can raise the scale of Lorentz violation well above even the electroweak scale, these bounds are relevant, but its easy to see that once we impose the experimental bound on μ, the astrophysical bound is almost automatically satisfied. For a pseudo-scalar φ coupling to the axial current, stellar energy loss arguments bound the coefficient of the operator

$$\frac{1}{F}J^5_\mu\partial^\mu\varphi \tag{8.30}$$

to be $F > 10^9$ GeV [58]. In the non-relativistic limit, Eq. (8.30) is identical to the canonically normalized coupling of the longitudinal mode of $\vec{\mathcal{A}}$ to spin-density, so as long as $M \gtrsim 10$ eV, the bound on μ is stricter than the bound on F directly. For the transverse modes of $\vec{\mathcal{A}}$, we can estimate the bound on M/F by going to canonical normalization and replacing the derivative ∂^μ with a typical astrophysical energy $E_{\rm astro} \sim 100$ keV:

$$\frac{1}{F}J^5_\mu\partial^\mu\varphi \sim \frac{E_{\rm astro}}{F}\vec{J}^5\cdot\vec{A}^c \qquad \Longrightarrow \qquad \frac{M}{F}g < 10^{-13}. \tag{8.31}$$

As long as $M \gtrsim 10^{-3}$ eV$/g$, the bound on μ enforces the astrophysical bound. This bound also applies to the vector coupling, requiring

$$\tilde{q}_\psi g < 10^{-13}. \tag{8.32}$$

Finally, for sources moving with respect to the preferred frame, there is both the possibility of Čerenkov radiation from gravitational couplings (see Sec. 7) and from the coupling in Eq. (8.16).[10] Because the scalar mode in gauged ghost condensation has a finite velocity, this Čerenkov effect is turned off when the velocity of the source v is smaller than c_s. When $v/c_s \gg 1$, we expect the discussion of Ref. [9] to go through virtually unchanged, where we found that within the range of validity of the effective theory, there were no observable effect from ether Čerenkov radiation for electrons. Ref. [59] studied the effect of Čerenkov radiation on neutrinos from SN1987A, where the bound was expressed as:

$$\left(\frac{M}{F}\right)_\nu < \frac{10^{-16}}{g^{3/2}}. \tag{8.33}$$

Assuming that neutrino couplings are comparable to electron couplings, this does not place any additional bounds on couplings between $\mathcal{A}_\mu$ and axial currents.

[10]Note that there is no Čerenkov radiation from the coupling in Eq. (8.5), because it only involves couplings to the transverse modes of A_i, which travel at c_v, *i.e.* nearly the speed of light.

9 Conclusions

We have analyzed spontaneous breaking of Lorentz symmetry by a vector condensate, a theory which gives rise to 3 extra modes in the gravitational sector. We showed that this theory can be viewed as a gauging of ghost condensation, the minimal theory of spontaneous breaking of Lorentz symmetry. The relation to ghost condensation allows us to understand the power counting of operators in the effective theory, and also to understand the limit where the gauge coupling g is small, where we recover ghost condensation. We also analyzed various constraints on this model, taking into account important nonlinear effects that are closely analogous to those found in ghost condensation [9].

We find that the limits on the scale M where Lorentz symmetry is spontaneously broken allow values much higher than the weak scale, suggesting that Lorentz violation may be more closely connected with fundamental particle physics. From purely gravitational constraints, we find that $M \lesssim \mathrm{Min}(10^{12}\ \mathrm{GeV},\, g^2\, 10^{15}\ \mathrm{GeV})$ from observation of a steller-mass black hole and constraints on the PPN parameters. If we allow all possible couplings to the standard model and cut off gravity loop effects at the scale M, we find that we need $M \lesssim 10^{13}$ GeV from the absence of observed Lorentz violation.

It is our hope that this work will bring Lorentz violation closer to the mainstream of particle physics, and that it will stimulate further investigations on the possible relation to the puzzles of gravity and cosmology.

A Decoupling Limit and Comparison with the Literature

Alternative theories of gravity with additional vector fields pointing in some preferred direction have been considered in the literature [11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22]. The value of the vector field is often imposed as a constraint through a Lagrange multiplier

$$\mathcal{L} \supset -\kappa(u^\mu u_\mu - 1). \tag{A.1}$$

We will show that this corresponds to certain decoupling limit of our theory. To see this, we can add a term $-2\kappa^2/M^4$ to the Lagrange multiplier. Integrating out κ we obtain

$$\mathcal{L} \supset \frac{M^4}{8}(u^2 - 1)^2 \tag{A.2}$$

which is exactly the leading term of our gauged ghost condensate Lagrangian:

$$\mathcal{L} \subset \frac{M^4}{8}\left(\frac{\mathcal{A}^\mu \mathcal{A}_\mu}{M^2} - 1\right)^2, \tag{A.3}$$

making the appropriate mapping between u^μ and $\mathcal{A}_\mu$. The constraint of the Lagrange multiplier is obtained by sending $M \to \infty$. Therefore, we see that these theories correspond to the decoupling limit $M \to \infty$ of gauged ghost condensation, with appropriate rescaling of other parameters, $g^{-2}, \alpha, \beta, \gamma, \ldots \to 0$ while keeping $g^{-2}M^2, \alpha M^2, \beta M^2, \gamma M^2, \ldots$ finite.

In this appendix we will study this decoupling limit and see the extent to which they are healthy modifications of gravity. We start with the ungauged ghost condensate with the lagrangain

$$\mathcal{L} = \mathcal{L}_{\mathrm{EH}} + \tfrac{1}{8}M^4(X-1)^2 - \tfrac{1}{2}\tilde{M}^2(\vec{\nabla}^2\pi)^2, \tag{A.4}$$

where $\mathcal{L}_{\mathrm{EH}}$ is the Einstein-Hilbert Lagrangian. For convenience we have defined $\tilde{M}^2 = \alpha M^2$. We want to understand the physics of taking $M \to \infty$, with $\tilde{M}$ and M_{Pl} held fixed. This is equivalent to imposing the constraint $X = 1$.

The linearized Lagrangian for the scalar sector including a matter source ρ is

$$\mathcal{L} = -M_{\mathrm{Pl}}^2(\vec{\nabla}\Phi)^2 + \tfrac{1}{2}M^4(\Phi - \dot{\pi})^2 - \tfrac{1}{2}\tilde{M}^2(\vec{\nabla}^2\pi)^2 + \rho\Phi. \tag{A.5}$$

For large M, we impose the constraint $\Phi = \dot{\pi}$, which gives the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = -M_{\mathrm{Pl}}^2(\vec{\nabla}\dot{\pi})^2 - \tfrac{1}{2}\tilde{M}^2(\vec{\nabla}^2\pi)^2 + \rho\dot{\pi}. \tag{A.6}$$

From this we easily read off the dispersion relation for the scalar mode:

$$\omega^2 = -\frac{\tilde{M}^2}{2M_{\mathrm{Pl}}^2}\vec{k}^2. \tag{A.7}$$

Note that this has the wrong sign for $\tilde{M}^2 > 0$, which is just a manifestation of the Jeans instability in ghost condensation. But for $\tilde{M}^2 < 0$, the scalar has a healthy dispersion relation.

Of course, the same results can be obtained from the dispersion relation for π with finite M, Rewriting in terms of $\tilde{M}$:

$$\omega^2 = -\frac{\tilde{M}^2}{2M_{\mathrm{Pl}}^2}\vec{k}^2 + \frac{\tilde{M}^2}{M^4}\vec{k}^4. \tag{A.8}$$

In the limit $M \to \infty$ only the first term survives. So the decoupling limit is equivalent to considering

$$|\vec{k}| \ll m = \frac{M^2}{\sqrt{2} M_{\text{Pl}}}. \tag{A.9}$$

Therefore, we can define a healthy theory as long as m is large enough to be a UV scale and if we change the sign of $\tilde{M}^2$ relative to the ghost condensate.

What is the Newtonian potential in this limit? Define the decoupling limit sound speed as

$$v_0^2 = -\frac{\tilde{M}^2}{2M_{\text{Pl}}^2}. \tag{A.10}$$

In the $|\vec{k}| \ll m$ limit, the $\Phi\Phi$ propagator is

$$\langle \Phi\Phi \rangle \to -\frac{1}{2M_{\text{Pl}}^2} \frac{1}{\vec{k}^2} \left(\frac{\omega^2}{\omega^2 - \vec{k}^2 v_0^2} \right). \tag{A.11}$$

At very late times ($\omega \to 0$), the theory does not have a Newtonian potential, so the decoupling limit of ghost condensation is not a viable modification of gravity at very late times. However, for times t short compared to r/v_0, $|\vec{k}| \gg \omega \gg v_0 |\vec{k}|$

$$\langle \Phi\Phi \rangle \to -\frac{1}{2M_{\text{Pl}}^2} \frac{1}{\vec{k}^2} + \mathcal{O}\left(\frac{\vec{k}^2 v_0^2}{\omega^2} \right), \tag{A.12}$$

and we recover an ordinary Newtonian potential plus small corrections. If v_0 is sufficiently small, then the decoupling limit could be a viable modification of gravity with interesting late time behavior, and we will study this limit in future work.

Now consider the gauged ghost condensate. To make the limit clearer, we define π to have mass dimension -1, A_μ to have mass dimension $+1$, and rescale the fields and couplings relative to Eq. (4.3) by

$$A_\mu \to \frac{M}{\mu} A_\mu, \quad \frac{1}{g^2} \to \left(\frac{\mu}{M} \right)^2 \frac{1}{g^2}, \tag{A.13}$$

where μ is a mass scale held fixed when we take the limit $M \to \infty$. The gauge transformations can now be written as

$$\delta A_\mu = \partial_\mu \chi, \qquad \delta\pi = -\frac{\chi}{\mu}, \tag{A.14}$$

and the scalar sector Lagrangian is

$$\mathcal{L} = -M_{\text{Pl}}^2 (\vec{\nabla}\Phi)^2 + \frac{1}{2g^2} (\vec{\nabla} A_0)^2 + \tfrac{1}{2} M^4 (\Phi - \dot{\pi} + A_0/\mu)^2 - \tfrac{1}{2} \tilde{M}^2 (\vec{\nabla}^2 \pi)^2. \tag{A.15}$$

We are interested in the limit $M \to \infty$ with the other couplings (including μ) held fixed. The dispersion relation for the scalar mode is

$$\omega^2 = -\frac{\tilde{M}^2}{2M_{\rm Pl}^2}\left(1 - \frac{2g^2 M_{\rm Pl}^2}{\mu^2}\right)\vec{k}^2 + \frac{\tilde{M}^2}{M^4}\vec{k}^4. \tag{A.16}$$

As before, the second term is absent in the limit $M \to \infty$, and is negligible for $|\vec{k}| \ll m$. The $\vec{k}^2$ term has the right sign for

$$\tilde{M}^2 > 0, \qquad g > g_{\rm c} = \frac{\mu}{\sqrt{2}M_{\rm Pl}}, \tag{A.17}$$

or for $\tilde{M}^2 < 0$, $g < g_{\rm c}$. It is easy to see that in order to get a viable Newtonian potential at very late times ($\omega \to 0$), we must choose the first option. The static limit of the $\Phi\Phi$ propagator is

$$\langle \Phi\Phi \rangle = -\frac{1}{2M_{\rm Pl}^2}\frac{1}{\vec{k}^2}\frac{1}{1 - g_{\rm c}^2/g^2}, \tag{A.18}$$

so if $g < g_{\rm c}$ this would have the wrong sign relative to the standard Newtonian potential. So as long as $g > g_c$, the decoupling limit of gauged ghost condensation is a healthy modification of gravity at leading order. Interestingly, if $\tilde{M}^2 < 0$ and $g < g_{\rm c}$, then there is once again an intermediate range of times for which there is an ordinary Newtonian potential.

This analysis shows why, for example, Ref. [33] finds finite corrections to $\alpha_2^{\rm PPN}$ in a model with the constraint in Eq. (A.1). In Sec. 4, we found

$$\alpha_2^{\rm PPN} \sim \frac{M^2}{\alpha g^4 M_{\rm Pl}^2}, \tag{A.19}$$

which would diverge as $M^2 \to \infty$. However in the decoupling limit where $g^{-2}M^2$ and αM^2 are held fixed, $\alpha_2^{\rm PPN}$ stays finite. We emphasize that while the decoupling limit is healthy, it obscures the power counting of gauged ghost condensation, which gives a systematic way to understand the relevance of operators in a Lorentz-violating setting.

B Stress-Energy Tensor of the Gauged Ghost Condensate

The goal of this appendix is to calculate the stress-energy tensor of the gauged ghost condensate. For this purpose we need to consider general variations of the metric since the stress-energy tensor is defined as the functional derivative of an action with respect to the metric components. On the other hand, in order to make the expression

independent of the heavy modes we have to integrate out the perturbation of $D_0\phi$ before taking the variation and, thus, we need to separate the time coordinate and space coordinates. For these reasons, we shall adopt the ADM decomposition of the metric:

$$ds^2 = -N^2 dt^2 + q_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \tag{B.1}$$

and define $\mathcal{A}_\mu$ by

$$D_0\phi = (1 + \mathcal{A}_0)N, \quad D_i\phi = \mathcal{A}_i. \tag{B.2}$$

We shall integrate $\mathcal{A}_0$ out.

Expanding $X - 1$ as

$$\frac{1}{2}(X-1) = \mathcal{A}_0 - \frac{1}{2}\left(q^{ij} - \frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_i\mathcal{A}_j + O(\mathcal{A}_0^2), \tag{B.3}$$

the leading Lagrangian is

$$\mathcal{L} = \frac{M^4}{2}\left[\mathcal{A}_0 - \frac{1}{2}\left(q^{ij} - \frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_i\mathcal{A}_j\right]^2 - \frac{M^2}{4g^2}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha M^2}{2}(\nabla^\mu D_\mu\phi)^2, \tag{B.4}$$

where $q^{ij} = (q^{-1})^{ij}$ and $q = \det q$. Integrating out the massive mode $\mathcal{A}_0$,

$$\mathcal{A}_0 = \frac{1}{2}\left(q^{ij} - \frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_i\mathcal{A}_j, \tag{B.5}$$

we obtain

$$F_{\mu\nu}F^{\mu\nu} = -2q^{ij}F_{\perp i}F_{\perp j} + q^{ik}q^{jl}F_{ij}F_{kl}, \tag{B.6}$$

and

$$\nabla^\mu D_\mu\phi = -\frac{1}{\sqrt{q}}\partial_\perp\left\{\sqrt{q}\left[1 - \frac{\beta^j\mathcal{A}_j}{N} + \frac{1}{2}\left(q^{kl} - \frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\right\} + \frac{1}{N\sqrt{q}}\partial_i(N\sqrt{q}q^{ij}\mathcal{A}_j), \tag{B.7}$$

where

$$\partial_\perp \equiv \frac{1}{N}(\partial_t - \beta^i\partial_i), \tag{B.8}$$

and

$$\begin{aligned} F_{\perp i} &\equiv \frac{1}{N}(F_{ti} - \beta^k F_{ki}) \\ &= \partial_\perp\mathcal{A}_i - \frac{1}{N}\partial_i\left\{N\left[1 + \frac{1}{2}\left(q^{kl} - \frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\right\} + \frac{\beta^k}{N}\partial_i\mathcal{A}_k, \end{aligned} \tag{B.9}$$

$$F_{ij} = \partial_i\mathcal{A}_j - \partial_j\mathcal{A}_i. \tag{B.10}$$

All components of the stress-energy tensor are given by taking the variation of the effective action w.r.t. N, β^i and q_{ij}. The result is

$$\begin{aligned}
T_{\perp\perp} &\equiv -\frac{1}{\sqrt{q}}\frac{\delta}{\delta N}\int dtd^3xN\sqrt{q}\mathcal{L} \\
&= \frac{M^2}{g^2}\left\{\frac{1}{2}q^{ij}F_{\perp i}F_{\perp j} - \left[1+\frac{1}{2}\left(q^{kl}+\frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\frac{1}{\sqrt{q}}\partial_i(\sqrt{q}q^{ij}F_{\perp j}) + \frac{1}{4}q^{ik}q^{jl}F_{ij}F_{kl}\right\} \\
&\quad +\alpha M^2\left\{-\frac{1}{2}(\nabla^\mu D_\mu\phi)^2 + \frac{\beta^i\mathcal{A}_i}{N}\left(1+\frac{\beta^j\mathcal{A}_j}{N}\right)\partial_\perp(\nabla^\mu D_\mu\phi) - q^{ij}\mathcal{A}_i\partial_j(\nabla^\mu D_\mu\phi)\right\}, \\
T_{\perp i} &\equiv -\frac{1}{\sqrt{q}}\frac{\delta}{\delta\beta^i}\int dtd^3xN\sqrt{q}\mathcal{L} \\
&= \frac{M^2}{g^2}\left[q^{jk}F_{ij}F_{\perp k} + \frac{\beta^j}{N}\mathcal{A}_i\mathcal{A}_j\frac{1}{\sqrt{q}}\partial_k(\sqrt{q}q^{kl}F_{\perp l})\right] \\
&\quad +\alpha M^2\left\{-\left(1+\frac{\beta^j\mathcal{A}_j}{N}\right)\mathcal{A}_i\partial_\perp(\nabla^\mu D_\mu\phi)\right. \\
&\quad \left. -\left[1-\frac{\beta^j\mathcal{A}_j}{N}+\frac{1}{2}\left(q^{kl}-\frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\partial_i(\nabla^\mu D_\mu\phi)\right\}, \\
T^{ij} &\equiv -\frac{2}{\sqrt{q}}\frac{\delta}{\delta q_{ij}}\int dtd^3xN\sqrt{q}\mathcal{L} \\
&= \frac{M^2}{g^2}\left\{\left(\frac{1}{2}q^{kl}F_{\perp k}F_{\perp l} - \frac{1}{4}q^{km}q^{ln}F_{kl}F_{mn}\right)q^{ij} - q^{ik}q^{jl}F_{\perp k}F_{\perp l} + q^{ik}q^{jl}q^{mn}F_{km}F_{ln}\right. \\
&\quad \left. -q^{ik}q^{jl}\mathcal{A}_k\mathcal{A}_l\frac{1}{\sqrt{q}}\partial_m(\sqrt{q}q^{mn}F_{\perp n})\right\} \\
&\quad +\alpha M^2\left\{\left[\frac{1}{2}(\nabla^\mu D_\mu\phi)^2\right.\right. \\
&\quad \left. -\left(1-\frac{\beta^k\mathcal{A}_k}{N}+\frac{1}{2}\left(q^{kl}-\frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right)\partial_\perp(\nabla^\mu D_\mu\phi) + q^{kl}\mathcal{A}_k\partial_l(\nabla^\mu D_\mu\phi)\right]q^{ij} \\
&\quad \left. +q^{ik}q^{jl}\mathcal{A}_k\mathcal{A}_l\partial_\perp(\nabla^\mu D_\mu\phi) - q^{ik}q^{jl}(\mathcal{A}_k\partial_l+\mathcal{A}_l\partial_k)(\nabla^\mu D_\mu\phi)\right\}.
\end{aligned} \tag{B.11}$$

The Einstein equation is

$$M_{\rm Pl}^2G_{\mu\nu}u^\mu u^\nu = T_{\perp\perp}, \quad M_{\rm Pl}^2G_{\mu i}u^\mu = T_{\perp i}, \quad M_{\rm Pl}^2G^{ij} = T^{ij}, \tag{B.12}$$

where

$$u^\mu = \left(\frac{\partial}{\partial t}\right)^\mu - \beta^i\left(\frac{\partial}{\partial x^i}\right)^\mu. \tag{B.13}$$

Just for completeness, the equation of motion for $\mathcal{A}_i$ is

$$\frac{1}{g^2}\left\{\frac{1}{\sqrt{q}}\partial_\perp(\sqrt{q}q^{ij}F_{\perp j}) - \left[\left(q^{ij}-\frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_j - \frac{\beta^i}{N}\right]\frac{1}{\sqrt{q}}\partial_k(\sqrt{q}q^{kl}F_{\perp l})\right.$$

$$+\frac{1}{N\sqrt{q}}\partial_j(N\sqrt{q}q^{ik}q^{jl}F_{kl})+\left(q^{kj}F_{\perp j}\frac{\partial_k\beta^i}{N}-q^{ij}F_{\perp j}\frac{\partial_k\beta^k}{N}\right)\Bigg\}$$

$$+\alpha\left\{\left[\left(q^{ij}-\frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_j-\frac{\beta^i}{N}\right]\partial_\perp(\nabla^\mu D_\mu\phi)-q^{ij}\partial_j(\nabla^\mu D_\mu\phi)\right\}=0. \qquad \text{(B.14)}$$

C Static Post-Newtonian Gravity

The PPN parameters β_{PPN} and γ_{PPN} are defined in the $1/r$-expansion of a spherically symmetric, static metric in the isotropic coordinate as

$$ds^2=-\left[1-\frac{2m}{r}+\frac{2\beta_{\text{PPN}}m^2}{r^2}+O(m^3/r^3)\right]dt^2+\left[1+\frac{2\gamma_{\text{PPN}}m}{r}+O(m^2/r^2)\right]\left(dr^2+r^2d\Omega_2^2\right). \qquad \text{(C.1)}$$

In this coordinate, β_{PPN} and γ_{PPN}, respectively, measure the amount of non-linearity and the amount of space curvature produced by a mass. The values in General Relativity is $\beta_{PPN}=\gamma_{PPN}=1$. Experimental limits on them are

$$|\beta_{\text{PPN}}-1|<10^{-3}, \quad |\gamma_{\text{PPN}}-1|<10^{-3}. \qquad \text{(C.2)}$$

In this section we calculate β_{PPN} and γ_{PPN} for the gauged ghost condensate.

For the isotropic coordinate

$$ds^2=-N(r)^2dt^2+B(r)^2\left(dr^2+r^2d\Omega_2^2\right), \qquad \text{(C.3)}$$

Einstein tensor is given by

$$\begin{aligned}
G_{\perp\perp}&=\frac{1}{B^2}\left[\left(\frac{B'}{B}\right)^2-\frac{4B'}{rB}-\frac{2B''}{B}\right],\\
G_{\perp r}&=G_{\perp\theta}=0,\\
G^{rr}&=\frac{1}{B^4}\left[2\frac{B'}{B}\frac{N'}{N}+\left(\frac{B'}{B}\right)^2+\frac{2}{r}\frac{B'}{B}+\frac{2}{r}\frac{N'}{N}\right],\\
G^{\theta\theta}&=\frac{1}{r^2B^4}\left[\frac{N''}{N}+\frac{N'}{rN}+\frac{B'}{rB}+\frac{B''}{B}-\left(\frac{B'}{B}\right)^2\right],\\
G^{r\theta}&=0.
\end{aligned} \qquad \text{(C.4)}$$

We shall apply the formulae obtained in App. B to this metric by setting

$$N=N(r), \quad \beta^i=0, \quad q_{ij}dx^idx^j=B(r)^2\left(dr^2+r^2d\Omega_2^2\right), \quad A_idx^i=A(r)dr. \qquad \text{(C.5)}$$

The result is

$$
\begin{aligned}
M^{-2}T_{\perp\perp} &= \frac{1}{g^2}\left[\frac{1}{2}\left(\frac{F_\perp}{B}\right)^2 - \left(1+\frac{A^2}{2B^2}\right)\frac{(r^2BF_\perp)'}{r^2B^3}\right] - \alpha\left[\frac{1}{2}(dA)^2 + \frac{A(dA)'}{B^2}\right],\\
M^{-2}T_{\perp r} &= -\alpha\left(1+\frac{A^2}{2B^2}\right)(dA)',\\
M^{-2}T_{\perp\theta} &= 0,\\
M^{-2}T^{rr} &= \frac{1}{B^2}\left\{-\frac{1}{g^2}\left[\frac{1}{2}\left(\frac{F_\perp}{B}\right)^2 + \frac{A^2(r^2BF_\perp)'}{r^2B^5}\right] + \alpha\left[\frac{1}{2}(dA)^2 - \frac{A(dA)'}{B^2}\right]\right\},\\
M^{-2}T^{\theta\theta} &= \frac{1}{r^2B^2}\left\{\frac{1}{2g^2}\left(\frac{F_\perp}{B}\right)^2 + \alpha\left[\frac{1}{2}(dA)^2 + \frac{A(dA)'}{B^2}\right]\right\},\\
M^{-2}T^{r\theta} &= 0,
\end{aligned}
\tag{C.6}
$$

where

$$
\begin{aligned}
F_\perp &= -\frac{1}{N}\left[N\left(1+\frac{A^2}{2B^2}\right)\right]',\\
dA &= \frac{(r^2NBA)'}{r^2NB^3}.
\end{aligned}
\tag{C.7}
$$

In order to calculate $\beta_{\rm PPN}$ and $\gamma_{\rm PPN}$, we expand all variables by $1/r$,

$$
\begin{aligned}
N(r) &= 1 + \sum_{n=1}^{\infty}\frac{N_n}{r^n},\\
B(r) &= 1 + \sum_{n=1}^{\infty}\frac{B_n}{r^n},\\
A(r) &= \sum_{n=0}^{\infty}\frac{A_n}{r^n}.
\end{aligned}
\tag{C.8}
$$

Accordingly, we obtain the $1/r$-expansion of the Einstein equation.

The $(\perp\, r)$-component of the Einstein equation is $\alpha(dA)' = 0$, which with the above $1/r$-expansion implies that $A = 0$. In this case non-vanishing components of the stress energy tensor are

$$
\begin{aligned}
M^{-2}T_{\perp\perp} &= \frac{1}{g^2}\left[\frac{1}{2}\left(\frac{N'}{NB}\right)^2 + \left(1+\frac{A^2}{2B^2}\right)\frac{1}{r^2B^3}\left(\frac{r^2BN'}{N}\right)'\right],\\
B^2M^{-2}T^{rr} &= -\frac{1}{g^2}\left[\frac{1}{2}\left(\frac{N'}{NB}\right)^2 + \frac{A^2}{r^2B^5}\left(\frac{r^2BN'}{N}\right)'\right],\\
r^2B^{-2}M^{-2}T^{\theta\theta} &= \frac{1}{2g^2}\left(\frac{N'}{NB}\right)^2.
\end{aligned}
\tag{C.9}
$$

The leading term in the (rr)-component of the Einstein equation says that

$$B_1 = -N_1. \tag{C.10}$$

With this relation, the leading term in $G_{\perp\perp} - B^2 G^{rr} = \kappa^2 (T_{\perp\perp} - B^2 T^{rr})$ is

$$(M^2 - 2g^2 M_{\rm Pl}^2)\left(N_2 - \frac{N_1^2}{2}\right) = 0. \tag{C.11}$$

Hence, unless $M^2 = 2g^2 M_{\rm Pl}^2$, we obtain

$$N_2 = \frac{N_1^2}{2}. \tag{C.12}$$

From (C.10) and (C.12) we obtain

$$\beta_{\rm PPN} = \gamma_{\rm PPN} = 1. \tag{C.13}$$

Therefore, there is no constraint from the spherically symmetric, static post-Newtonian gravity. The leading correction to General Relativity appears in B_2/N_1^2. Actually, the leading term in the $(\perp\perp)$-component of the Einstein equation says that

$$\frac{B_2}{N_1^2} = \frac{1}{4}\left(1 + \frac{M^2}{2g^2 M_{\rm Pl}^2}\right), \tag{C.14}$$

and this value of B_2/N_1^2 has a deviation from the GR value 1/4. However, this deviation is in the post-post Newtonian order and beyond the current ability of gravity experiments.

D Fragmentation of Planar Caustics

In Sec. 5 we have provided both analytical and numerical evidences showing that there should be no instability except for the perfectly plane symmetric case. We have also pointed out that, on the other hand, it is in principle possible to create a caustic plane if the perfectly planar symmetry is assumed. However, this situation is unlikely to happen in generic situations since the perfect plane symmetry is too ideal an assumption. Indeed, small fluctuations on top of the plane symmetric collapse should grow and the layer should be broken into pieces before a planar caustics occurs. After the fragmentation, a lower-dimensional caustics does not occur since, as discussed in subsection 5.3, for codimension 2 or higher it would cost infinite amount of work to compress a finite-volume, negative $r^s E_r$ $(s = 1, 2)$ region to an infinitesimal volume or thickness.

In this appendix we perform a perturbative analysis supporting this picture of fragmentation. In particular, we analyze inhomogeneous linear perturbation on top of a perfectly planer caustic solution and show that inhomogeneous perturbation grows faster than the background caustics.

In a fixed flat spacetime background $ds^2 = dt^2 - dx^2 - dz^2$, we consider a configuration depending on two space coordinates x and z and the time t:

$$A_i dx^i = A_x(t,x,z)dx + A_z(t,x,z)dz. \tag{D.1}$$

The equation of motion is then

$$\begin{aligned} \dot{E}_i &= A_i \partial^j E_j - \partial^j F_{ij} + \alpha g^2 \partial_i \partial^j A_j, \\ E_i &\equiv \dot{A}_i - A^j \partial_i A_j, \end{aligned} \tag{D.2}$$

where i and j run over x and z. We consider a planar scaling solution with inhomogeneous linear perturbation of the form

$$A_z = \frac{z}{-t}\left[1 + f(t)e^{ikx}\right], \quad A_x = g(t)e^{ikx}, \tag{D.3}$$

where we consider f and g as small perturbations. Since we are interested in the behavior near the caustic plane $z = 0$, we have truncated the Taylor expansion w.r.t. z at the lowest order. In the first order in f and g, the equation of motion becomes

$$\begin{aligned} \ddot{f} - \frac{\dot{f}}{-t} + \left[k^2 - \frac{k^2 z^2 + 1}{(-t)^2}\right] f + ik\left[\frac{\dot{g}}{-t} + \frac{g}{(-t)^2}\right] &= 0, \\ \ddot{g} + \frac{2\dot{g}}{-t} + \frac{2g}{(-t)^2} + ikz^2 \frac{\dot{f}}{-t} + ik\left[\frac{2z^2}{(-t)^2} - 1\right] f &= 0, \end{aligned} \tag{D.4}$$

By eliminating g we obtain the third order equation for f as

$$\dddot{f} - \frac{\ddot{f}}{-t} + \left[k^2 - \frac{2}{(-t)^2}\right]\dot{f} - \left[k^2 + \frac{2}{(-t)^2}\right]\frac{f}{-t} = 0. \tag{D.5}$$

The general solution to this equation is

$$f = \frac{C_1}{-t} + C_2\left[k\cos(kt) + \frac{1}{-t}\sin(kt)\right] + C_3\left[k\sin(kt) - \frac{1}{-t}\cos(kt)\right], \tag{D.6}$$

where C_1, C_2 and C_3 are constants. Hence, the linear perturbation is unstable and grows faster than the background.

E A Simple Way to Calculate the Energy Loss in Čerenkov Radiation

Consider a Lagrangian in the presence of an external source

$$\mathcal{L}_{\text{Total}} = \mathcal{L}_\phi(\phi,\, \partial\phi) + \mathcal{L}_S(\phi,\, S), \tag{E.1}$$

where $\mathcal{L}_S$ represent the source term, *e.g.* $\mathcal{L}_S = S\phi$. If the action without the source $\mathcal{S}_\phi = \int d^4x \mathcal{L}_\phi$ is invariant under the transformation $\phi \to \phi' = \phi + \delta\phi$, there is an associated conserved Noether current

$$J^\mu_\phi = \frac{\delta\mathcal{L}_\phi}{\delta(\partial_\mu\phi)}\delta\phi - V^\mu, \tag{E.2}$$

with V^μ given by

$$\delta\mathcal{L}_\phi = \partial_\mu V^\mu. \tag{E.3}$$

The presence of the (fixed) external source "breaks" the invariance of the transformation, so the current is no longer conserved:

$$\partial_\mu J^\mu_\phi = \delta\mathcal{L}_S. \tag{E.4}$$

Integrating over space, we obtain

$$\int d^3x\, \delta\mathcal{L}_S = \int d^3x\, \partial_\mu J^\mu_\phi = \partial_0 Q_\phi + \oint J^i dS_i. \tag{E.5}$$

The right hand side is just the total charge flowing into the ϕ field and out to infinity, so it must be compensated by the loss of the total charge of the source, $-dQ_S/dt$:

$$\frac{dQ_S}{dt} = -\int d^3x\, \delta\mathcal{L}_S. \tag{E.6}$$

For energy, the corresponding symmetry transformation is time translation, so $\delta\phi = \dot{\phi}$. The energy loss for a source $\mathcal{L}_S = S\phi$ is then simply given by

$$\frac{dE_S}{dt} = -\int d^3x\, S\, \dot{\phi}. \tag{E.7}$$

In the linearized approximation, $\dot{\phi}$ can be easily solved in terms of a given source in momentum space. It is then straightforward to compute the energy loss using Eq. (E.7). It applies to both the Čerenkov radiation when a source moving in a medium faster than the sound speed, and multipole radiations due to accelerations. As an example, we derive the energy loss from the Goldstone boson radiation in a binary system. The results are used in sec. 7 to derive bounds on the scale of the Lorentz violation from the binary pulsars.

We consider a system of binary pulsars where two pulsars of equal mass M_0 move in a circular orbit separated by a distance $2r_0$. We ignore the velocity of the center of the system for this calculation.

In the δ-function approximation, the source is given by

$$\rho(x,t) = M_0\delta^3\left(\vec{x} - \vec{r}(t)\right) + M_0\delta^3\left(\vec{x} + \vec{r}(t)\right), \tag{E.8}$$

$$\vec{r}(t) = (r_0\cos\omega_0 t,\ r_0\sin\omega_0 t,\ 0), \tag{E.9}$$

where the angular frequency ω_0 is related to the orbital velocity v_0 by $v_0 = \omega_0 r_0$. Its Fourier transform is

$$\tilde{\rho}(\omega,k) = 2M_0\sum_{n=-\infty}^{\infty} e^{i\,2n(\theta_k - \frac{\pi}{2})} J_{2n}(k_{\parallel}r_0)\,2\pi\delta(\omega - 2n\omega_0), \tag{E.10}$$

where $k_{\parallel} = \sqrt{k_x^2 + k_y^2}$, $\tan\theta_k = k_y/k_x$, and we have used the property of the Bessel functions, $J_m(-x) = (-1)^m J_m(x)$. A form factor $f(k)$ can also be included to represent the finite size effect. Applying the simple way of calculating the energy loss described above and using Eqs.(7.1)–(7.3), we obtain the time-averaged energy loss rate

$$\frac{dE_S}{dt} = -\frac{\alpha M_0^2 M^2}{4\pi M_{\rm Pl}^4 c_s}\sum_{n=-\infty}^{\infty}\int_0^1 d(\cos\theta)\left(\frac{2n\omega_0}{c_s}\right)^2 J_{2n}\left(\frac{2n\omega_0 r_0\sin\theta}{c_s}\right)^2\left|f\left(\frac{2n\omega_0}{c_s}\right)^2\right|. \tag{E.11}$$

Now we can consider two different limits. First, for $v_0 = \omega_0 r_0 \ll c_s$ we can use the approximation

$$J_m(x) \simeq \frac{1}{m!}\left(\frac{x}{2}\right)^m, \quad \text{for small } x. \tag{E.12}$$

The sum is dominated by the smallest $|n|$, $n = \pm 1$ which corresponds to quadrupole radiation. There is no form factor suppression ($f(k) \simeq 1$) because the inverse of the momentum is much bigger than the size of the system. The resulting energy loss by the quadrupole radiation is

$$\frac{dE_S}{dt} = -\frac{4\alpha M_0^2 M^2 v_0^6}{15\pi M_{\rm Pl}^4 r_0^2 c_s^7} = -\frac{2^{12}\pi\alpha M^2 v_0^{10}}{15\,c_s^7}, \tag{E.13}$$

where in the last equality we have used the relation $G_N M_0/r_0 = 4v_0^2$.

In the opposite limit, $c_s \ll v_0$, we need to use the asymptotic approximation of the Bessel functions,

$$J_m(x) \simeq \sqrt{\frac{2}{\pi x}}\cos\left[x - \left(m + \frac{1}{2}\right)\frac{\pi}{2}\right], \quad \text{for } x \gg 1. \tag{E.14}$$

From Eq. (E.11) the energy loss is given approximately by

$$\frac{dE_S}{dt} \simeq -\frac{\alpha M^2 M_0^2}{4\pi M_{\rm Pl}^4 c_s r_0^2} \sum_{n=1}^{\infty} \frac{2nv_0}{c_s} \left| f\left(\frac{2nv_0}{c_s r_0}\right)^2 \right|. \tag{E.15}$$

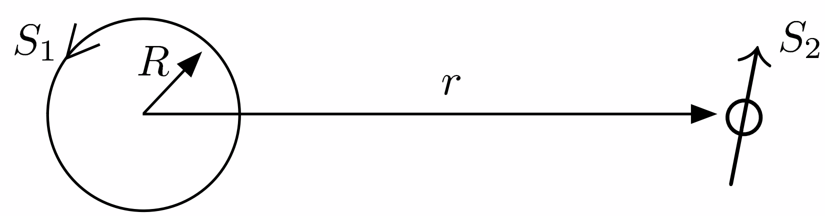
F A Null Test for a Spin-Dependent Force Law

It is well known that in electromagnetism, the magnetic field outside of a divergence-free spin configuration vanishes. Similarly for (ungauged) ghost condensation in the non-relativistic limit, the coupling between spin and ghostone boson field π takes the form [9]

$$\mathcal{L}_{\rm int} = \frac{M}{F} \vec{s} \cdot \vec{\nabla}\pi, \tag{F.1}$$

and doing an integration by parts, we see that π is not sourced by a divergence-free spin configuration. However, the potential in Eq. (8.26) is not of the form of Eq. (F.1). In particular, the transverse modes of $\vec{A}_i$ couple directly to $\vec{s}$ and not to its divergence.

One interesting divergence-free spin configuration is a ring of spin. This could be established with a toroidal electromagnet, and because the magnetic field is zero outside of the torus, this allows a null test for the gauged ghost condensate spin potential. Consider the following setup, where a ring of radius R with net spin S_1 is separated a distance r away from a point spin S_2.



(F.2)

It is straightforward to calculate the potential from Eq. (8.26):

$$V(r) = \frac{M^2 g^2}{F^2} \frac{S_1 S_{2y}}{8\pi r} \frac{R}{r} + \mathcal{O}(R^3/r^3), \tag{F.3}$$

where S_{2y} is the projection of S_2 in the vertical direction. Note that this potential does not depend on α, because α only controls the coupling of the longitudinal mode of $\vec{A}$. Though this potential goes as $1/r^2$, there is in principle no magnetic field leakage from the ring S_1, so it should be easier to design a null experiment to look for this long-range spin-spin potential.

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